# Verifiability in Communication
## A (Prospective) Experimental Analysis

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very preliminary

# Overview

An experimental study on the role of **verifiability in communication**

- Communication is classic problem in economics
- Verifiability (or lack thereof) is a core ingredient in our theories of communication

**This Paper:**

- Flexible framework to introduce rich variations in verifiability
- Develop novel comparative statics that inform experimental design
- Test main qualitative prediction of theory against observed subjects' behavior

model

# A Sender-Receiver Game

We build on the communication model by Milgrom (1981):

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   - $\theta \in \Theta$ with common prior $p \in \Delta(\Theta)$

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1. Sender privately observes the state $\theta$:
   - $\theta \in \Theta$ with common prior $p \in \Delta(\Theta)$
2. Given $\theta$, Sender draws $N \in \mathbb{N}$ signals from:
   - An information structure $f : \Theta \to \Delta(\Omega)$
   - $N$ conditionally independent draws from $f(\cdot|\theta)$

   Notation: $\bar{\omega} = (\omega_1, \ldots, \omega_N) \in \Omega^N$ sender's "*type*"

We build on the communication model by Milgrom (1981):

3. Sender verifiably discloses at most $K$ of her $N$ signals
   - Given $\bar{\omega}$, sender chooses $m \in M(\bar{\omega})$:

$$M(\bar{\omega}) := \Big\{ m \in \Omega^k \mid k \leq K \text{ and } \exists \text{ injective } \rho : \{1, ..., k\} \to \{1, ..., N\} \text{ s.t. } m = (\omega_{\rho(1)}, ..., \omega_{\rho(k)}) \Big\} \cup \{\varnothing\}.$$

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4. Receiver observes $m$, takes an action, payoff realizes
   - Receiver's action $a \in A$ and payoffs are:

$$u_S(\theta, a) = a \qquad u_R(\theta, a) = -(a - \theta)^2$$

**Summary:** Three parameters of interests

- $N$, the number of verifiable signals
- $K$, the number of reportable signals
- $f$, the information structure

**Assumptions:**

- $\Theta$ and $\Omega$ finite subsets of $\mathbb{R}$; $A = \mathbb{R}$
- $f$ satisfies **MLR property**: For $\theta' > \theta$, $\frac{f(\omega|\theta')}{f(\omega|\theta)}$ strictly increasing in $\omega$

**Applications:** News Media; VC financing; etc.

Interpretations $K < N$: institutional norm, attention cost

# discussion

# A Simple Example



- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$
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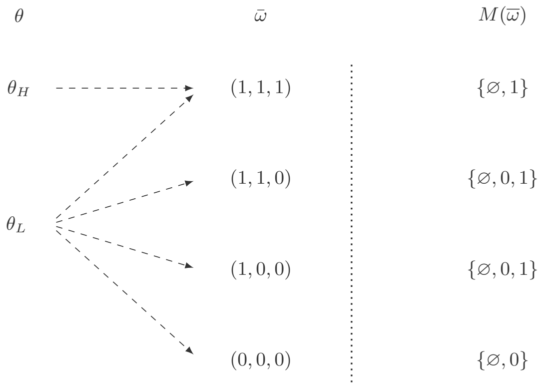

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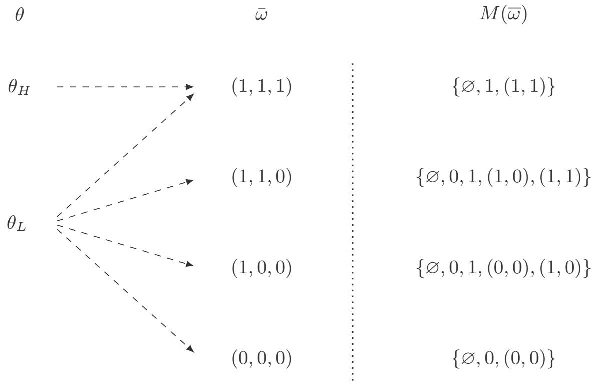

# Discussion



A model of **partially verifiable** information:

- When $K = N$, $\bar{\omega} \in M(\bar{\omega})$, ubiquitous assumption $\rightsquigarrow$ unravelling
- When $K < N$, $\bar{\omega} \notin M(\bar{\omega})$, Sender can only prove so much about herself: scope for imitation via **selective disclosure**

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Hybrid framework btw **cheap-talk** games and **disclosure** games

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Hybrid framework btw **cheap-talk** games and **disclosure** games

Changing $K$ and $N$ affects degree to which information is verifiable

- $N \uparrow$, Sender draws more verifiable signals about her type
- $K \uparrow$, Sender can report more signals to receiver

# This Paper

Verifiability (or lack thereof) is a fundamental ingredient of communication

Changing $K$ and $N$ is tool to introduce variation in degree of verifiability

Generates rich and asymmetric comparative statics, which inform our experimental design

Test qualitative predictions against observed behavior

Question overlooked by experimental literature

Closest papers that feature partially verifiable information, $\bar{\omega} \notin M(\bar{\omega})$

### The Basic Setting:

- Milgrom (1981, Bell), example to showcase MLRP
- Fishman and Hagerty (1990, QJE), optimal amount of discretion

### Mechanism-Design Approach:

- Glazer and Rubinstein (2004, Ecma) – Receiver's Verification, $K = 1$
- Glazer and Rubinstein (2006, TE) – Sender's verification

### Richer Settings: Uknown $N$ or Endogenous $K$

- Shin (2003, Ecma)
- Dziuda (2011, JET)

Rich experimental literature on communication

### Cheap Talk:

- Cai and Wang (2006, GEB) – overcommunication wrt the theory

### Verifiable Disclosure:

- Hagenbach and Perez-Richet (2018, GEB) – preference alignment
- Li and Schipper (2020, GEB) – vague disclosure
- Jin, Luca and Martin (2022, AEJ: Micro) – failure of unravelling

### Partially Verifiable Disclosure

- Burdea, Montero, Sefton (2022) – test of Glazer, Rubinstein ('04, '06)
- Li and Schipper (2018) – asymmetric info on amount of evidence
- Penczynski, Koch and Zhang (2021) – private acquisition of evidence

equilibrium

**Solution Concept:** Sequential Equilibrium details

**Proposition (Existence)** Milgrom (1981)

For all $N$ and $0 \leq K \leq N$, a sequential equilibrium exists where sender reports the $K$ **most favorable** signals in $\bar{\omega}$.

Two predictions about sender's behavior:

1. Sender discloses $K$ signals
2. When $K < N$, sender discloses most favorable signals

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Two predictions about sender's behavior:

1. Sender discloses $K$ signals "quantity"
2. When $K < N$, sender discloses most favorable signals "quality"

# Back to Example

- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$
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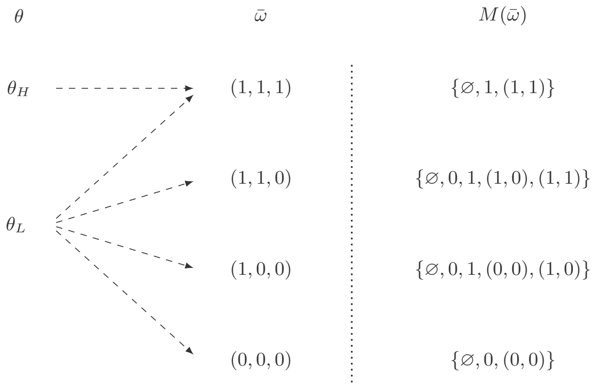

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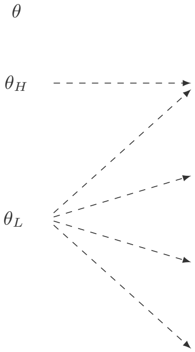


| $\bar{\omega}$ | $M(\bar{\omega})$ | $\sigma^*(\bar{\omega})$ |
|---|---|---|
| $(1,1,1)$ | $\{\varnothing, 1, (1,1)\}$ | $(1,1)$ |
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Off-path beliefs can support other equilibrium outcome

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| $\theta_H$ | $(1,1,1)$ | $\{\varnothing, 1\}$ | $\varnothing$ |
| | $(1,1,0)$ | $\{\varnothing, 0, 1\}$ | $\varnothing$ |
| $\theta_L$ | **$(1,0,0)$** | $\{\varnothing, 0, 1\}$ | $\varnothing$ |
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- Refinements for signalling games (e.g., Cho-Kreps ’87, Banks-Sobel ’87) have no force here
- Refinements for cheap talks: Farrel (1993)’s **Neologism Proofness**, Matthews, Okuno-Fujiwara, Postelwite (1991), and some weaker versions

Go to Credible Neologism

**Remark (Existence, again)**

Our equilibrium is Neologism Proof.

Moreover, Neologism Proofness delivers outcome uniqueness

An equilibrium $(\sigma, \mu)$ induces an outcome $x : \Omega^N \to A$,

$$x(\bar{\omega}) = \sum_{\bar{\omega}'} \mu(\bar{\omega}' | \sigma(\bar{\omega})) \mathbb{E}(\theta | \bar{\omega}') \qquad \forall \bar{\omega}.$$

**Proposition (Uniqueness)**

Let $(\sigma^*, \mu^*)$ be our equilibrium and $(\sigma, \mu)$ be any other NP equilibrium. Let $x^*$ and $x$ their respective outcomes. Then, $x^* = x$.

verifiability and communication

How does an increase in $K$ affect information transmission?

- $M(\bar{\omega})$ becomes larger $\Rightarrow$ Easier to send messages that others cannot imitate

**Proposition**

Fix $N$ and $f$ and $K < K' \leq N$. The equilibrium under $K'$ is Blackwell more informative than under $K$.

Proof shows that equilibrium partition $\{\sigma^{*-1}(m)\}_{m \in \sigma^*(\Omega^N)} \subseteq \Omega^N$ becomes finer as $K$ increases

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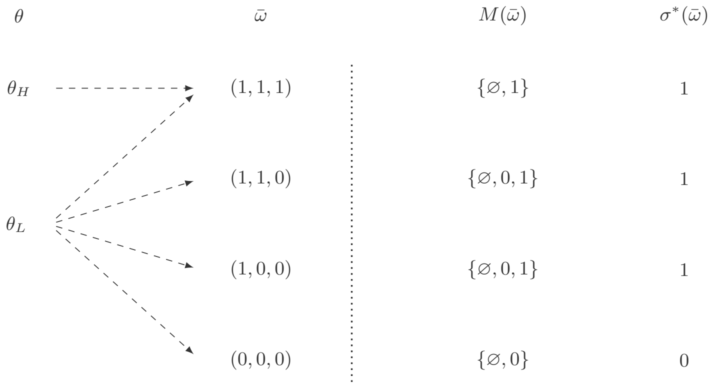

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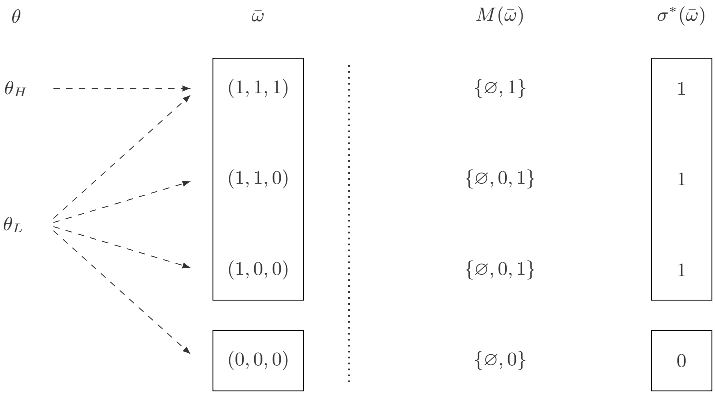

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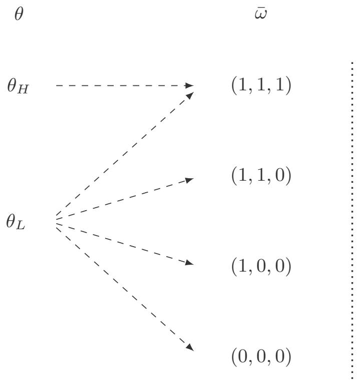

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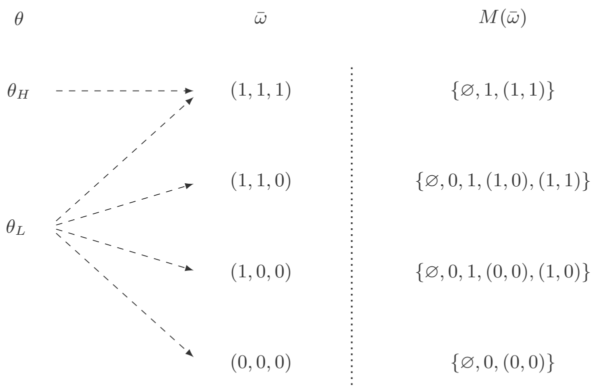

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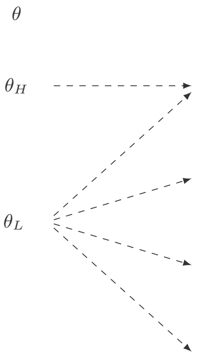


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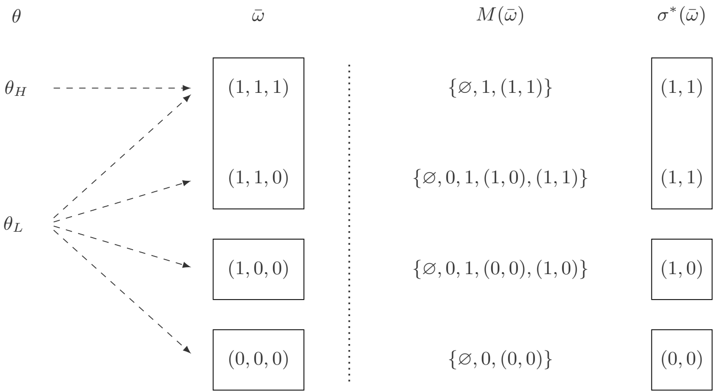

# Increasing $N$

How does an increase in $N$ affect information transmission?

A nontrivial tradeoff:

+ Higher $N$ $\rightsquigarrow$ sender is endowed with more verifiable signals
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**Today:**

- Results for two special (but interesting) cases
- A conjecture for the general case

# Case 1: Increasing $N$ is Bad



- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$; $K = 1$
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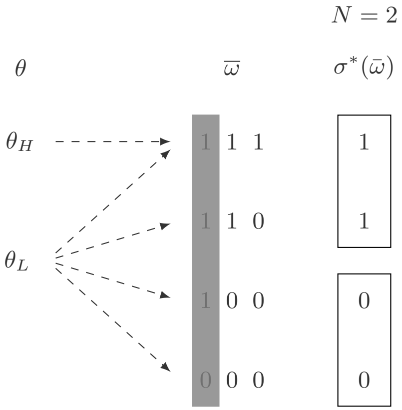

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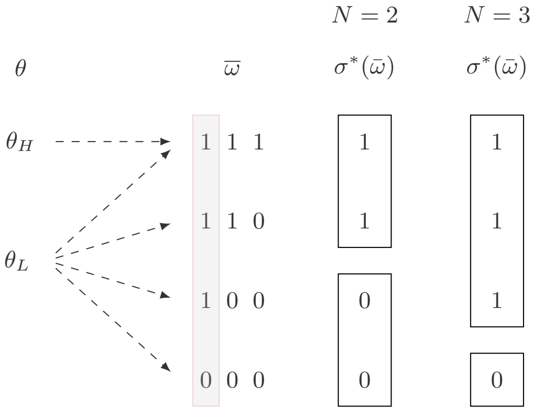

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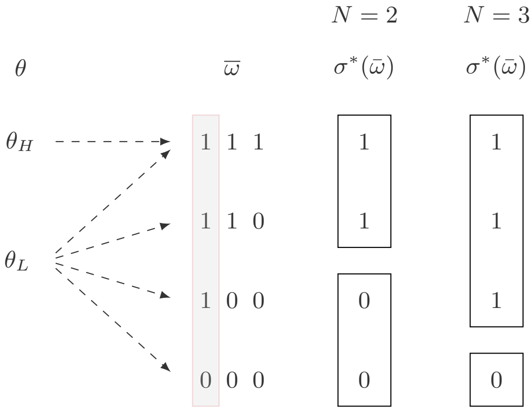


$N \uparrow \rightsquigarrow$ more likely that $\theta_L$ is able to imitate $\theta_H \rightsquigarrow$ informativeness $\downarrow$

# Case 2: Increasing $N$ is Good



- Binary state $\Theta = \{\theta_L, \theta_H\}$ and binary signals $\Omega = \{0, 1\}$, $K = 1$
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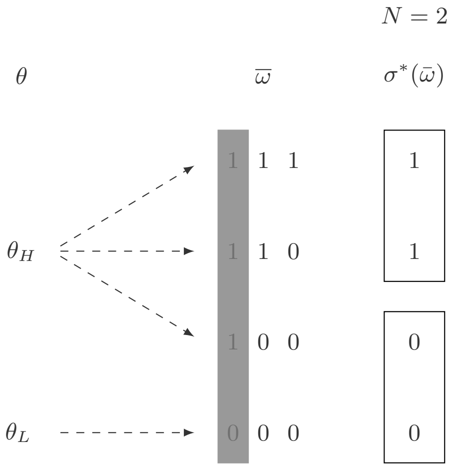

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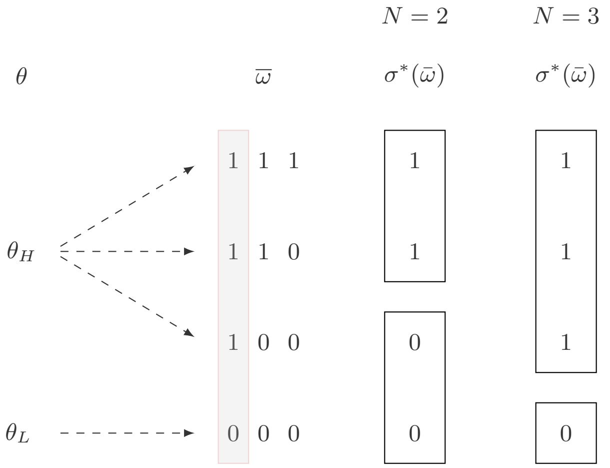

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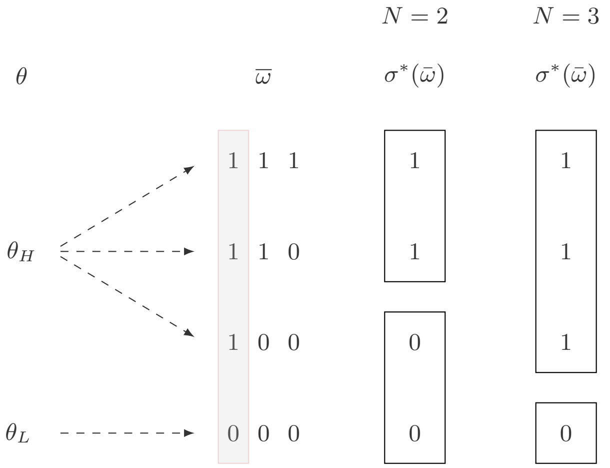


$N \uparrow \rightsquigarrow$ less likely that $\theta_L$ is able to imitate $\theta_H \rightsquigarrow$ informativeness $\uparrow$

# Comparative Statics

When $f$ is conclusive, we obtain the following comparative statics:

**Proposition**

Let $\Theta$ and $\Omega$ be binary. Fix any $K$. As $N$ increases, the receiver's payoff

- Increases if $f$ has conclusive good news
- Decreases if $f$ has conclusive bad news

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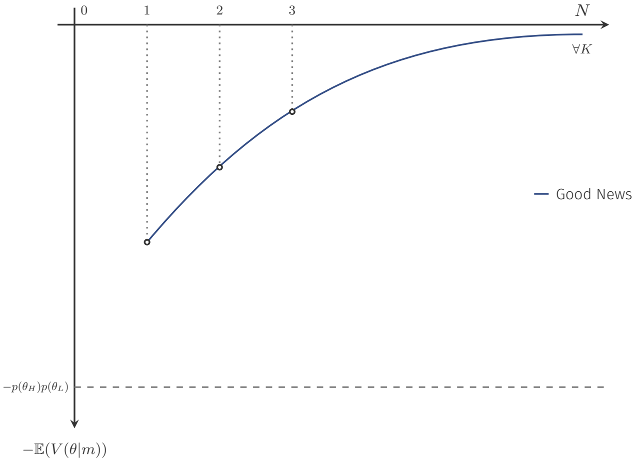

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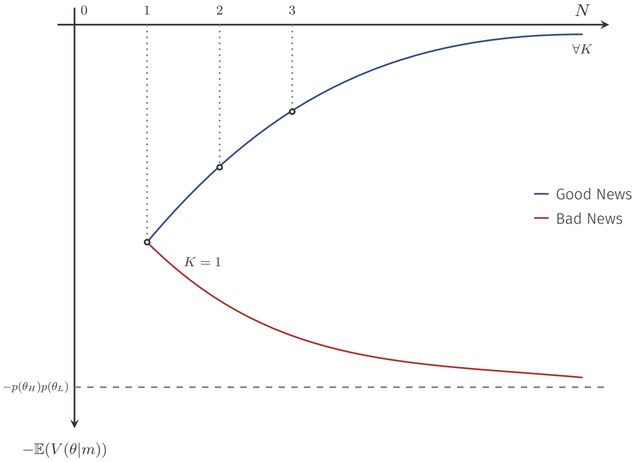

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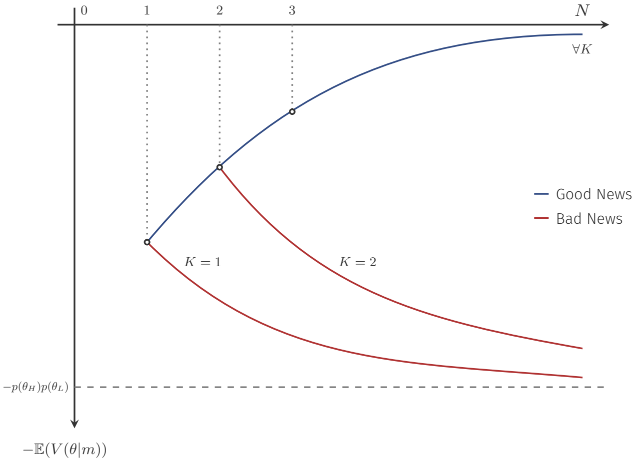

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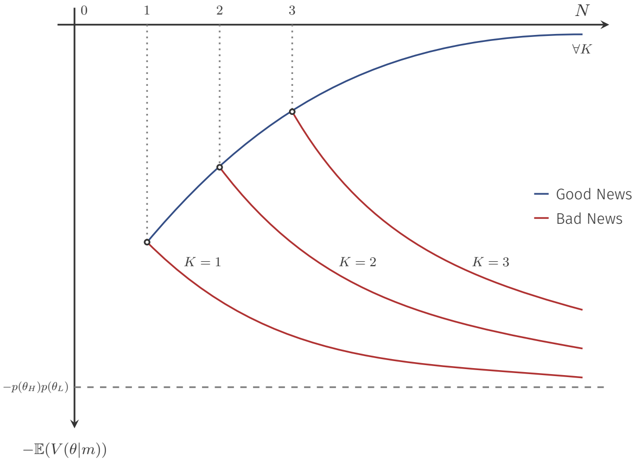

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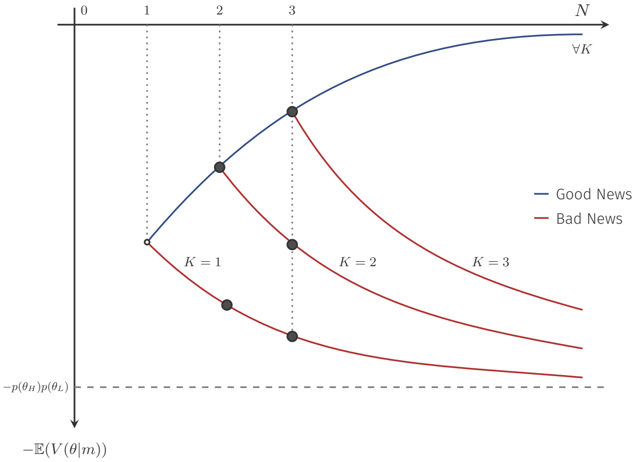


Experimental Design: Discussion of pros and cons

# What About the General Case?



Towards a more general result for the effects of changing $N$:

Binary state $\theta$ $+$ Binary signals $\omega$ $+$ Conclusive good/bad news $f$

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| | | | | |
|---|---|---|---|---|
| ~~Binary state $\theta$~~ | + | ~~Binary signals $\omega$~~ | + | ~~Conclusive good/bad news $f$~~ |
| Generic state $\theta$ | | Generic signals $\omega$ | | Generic $f$ |

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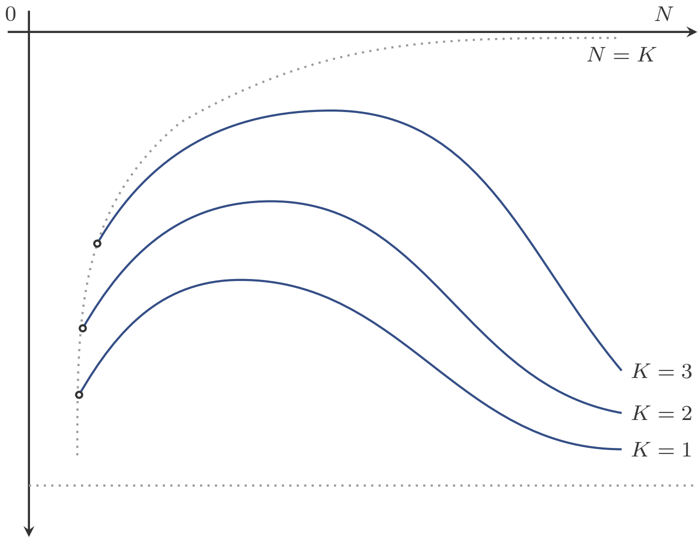
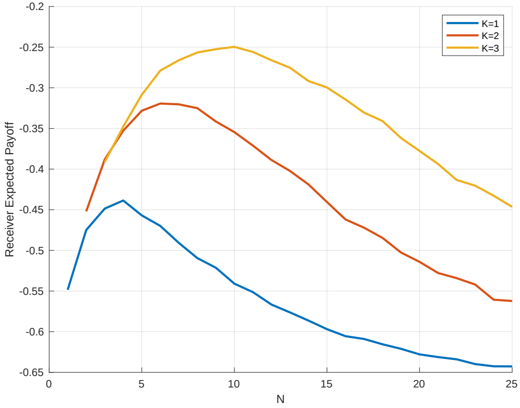
| Binary state $\theta$ | + | Binary signals $\omega$ | + | Conclusive good/bad news $f$ |
|---|---|---|---|---|
| Generic state $\theta$ | | Generic signals $\omega$ | | Generic $f$ |

**Conjecture**

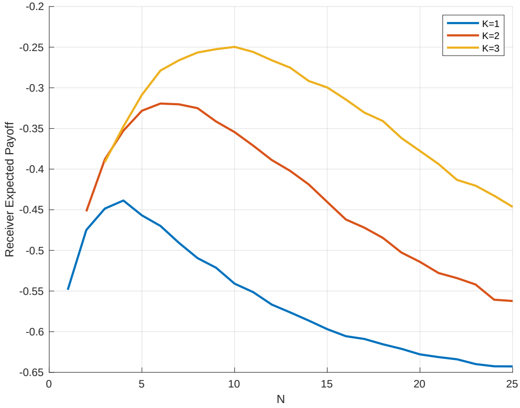
Fix $K$ and $f$. There exists $N^\star \in \mathbb{N} \cup \{\infty\}$ such that informativeness increases up until $N^\star$ and decreases afterwards



0
$N$
$N = K$
$K = 3$
$K = 2$
$K = 1$



K=1
K=2
K=3
Receiver Expected Payoff
N
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
-0.6
-0.65
0
5
10
15
20
25




Experimental Design: Discussion of pros and cons

# Other Directions



The framework is conducive to analyze more questions:

- Partial verifiability enables study of preference alignement:

  Verifiability helps info transmission when preferences misaligned

  Verifiability may hurt info transmission when preferences aligned

- Costly $N$

- Ex-ante disclosure

# summing up

# Summary



Verifiability is a key ingredient in communication

Flexible framework to introduce rich variations in verifiability

Stark comparative statics inform our experimental design

Test main qualitative predictions of the theory against observed subjects behavior (contrast with literature)

- If confirmed, this is empirical validation for a core component of our theories of communication
- If not, it indicates something off in our theories

thank you

# appendix

# Strategies and Beliefs

Denote $\mathcal{M} = \bigcup_{\bar{\omega} \in \Omega^N} M(\bar{\omega})$ the space of all messages

## Sender's Strategy

pure and $\theta$-independent

- $\sigma : \Omega^N \to \mathcal{M}$ s.t. $\sigma(\bar{\omega}) \in M(\bar{\omega})$, for all $\bar{\omega}$

## Receiver's Beliefs and Strategy

- $\mu : \mathcal{M} \to \Delta(\Omega^N)$
- $a : \mathcal{M} \to \Delta(A)$

Given $\mu$, receiver's optimal strategy given by

$$a(m) := \arg\max_a \mathbb{E}(-(a-\theta)^2 | m) = \mathbb{E}(\theta | m)$$

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**Definition:**

A Sequential Equilibrium is a pair $(\sigma^*, \mu^*)$ s.t.

1. For all $\bar{\omega} \in \Omega^N$, $\sigma^*(\bar{\omega}) \in M(\bar{\omega})$ and

$$\sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|\sigma^*(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \geq \sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|m')\mathbb{E}(\theta|\bar{\omega}') \qquad m' \in M(\bar{\omega})$$

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$\tilde{C}(m) := \{\bar{\omega} \in \Omega^N : m \in M(\bar{\omega})\}$; types that could have sent $m$

# Equilibrium

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2. For all $m$, supp $\mu^*(\cdot|m) \subseteq \tilde{C}(m)$. In particular, if $m \in \sigma^*(\Omega^N)$,

$$\mu^*(\bar{\omega}|m) = q(\bar{\omega}|\sigma^{\star^{-1}}(m)) \quad \forall \bar{\omega}$$

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Total Prob: $q(\bar{\omega}) = \sum_\theta p(\theta) f(\bar{\omega}|\theta)$; Conditional Prob: $q(\bar{\omega}|K)$

We refine off-path beliefs via **Neologism Proofness** (Farrel, 1993)

A **neologism** is a a pair $(m, C)$ such that $C \subseteq \tilde{C}(m)$.

Literal meaning of $(m, C)$ $\rightsquigarrow$ "*My type $\bar{\omega}$ belongs to $C$*"

# Neologism Proofness

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Literal meaning of $(m, C)$ $\rightsquigarrow$ "*My type $\bar{\omega}$ belongs to $C$*"

### Definition

A neologism $(m, C)$ is **credible** relative to equilibrium $(\sigma^*, \mu^*)$ if

$$(i) \quad \sum_{\bar{\omega}'} q(\bar{\omega}'|C)\mathbb{E}(\theta|\bar{\omega}') > \sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|\sigma^*(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \text{ for all } \bar{\omega} \in C,$$

$$(ii) \quad \sum_{\bar{\omega}'} q(\bar{\omega}'|C)\mathbb{E}(\theta|\bar{\omega}') \leq \sum_{\bar{\omega}'} \mu^*(\bar{\omega}'|\sigma^*(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \text{ for all } \bar{\omega} \notin C,$$

The equilibrium is **neologism proof** if no neologism is credible

## Remark (Existence, again)

Our equilibrium is Neologism Proof.

Moreover, Neologism Proofness delivers outcome uniqueness

An equilibrium $(\sigma, \mu)$ induces an outcome $x : \Omega^N \to A$,

$$x(\bar{\omega}) = \sum_{\bar{\omega}'} \mu(\bar{\omega}'|\sigma(\bar{\omega}))\mathbb{E}(\theta|\bar{\omega}') \qquad \forall \bar{\omega}.$$

## Proposition (Uniqueness)

Let $(\sigma^*, \mu^*)$ be our equilibrium and $(\sigma, \mu)$ be any other NP equilibrium. Let $x^*$ and $x$ their respective outcomes. Then, $x^* = x$.

In lab implementation, we may not need all these neologisms in:

$$\{(m, C) : m \in \mathcal{M}, C \subseteq \tilde{C}(m)\}$$

When $\Omega$ is binary, it is sufficient to consider these neologisms:

If $m$ is off-path its literal meaning is "*my highest $k$ signals are $m$*"

**Proposition**

If $\Omega$ is binary, weaker refinement guarantees outcome uniqueness.