Belief Meddling in Social Networks: an Information-Design Approach

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October 2018

Motivation

- 2012 Presidential race, Romney at **private** meeting with campaign donors: "47 percent of voters receive government money and feel entitled to that. They will never vote for me..."
- Recorded on video and leaked. Possible cause of defeat.
- Maybe right campaign message for audience in that private room. Not for general public.
- Privacy is key and also hard to enforce $\Rightarrow$ information spillovers

Overview

Pervasive issue when information is tool for influencing behavior.

- Information is easily replicable and non-exclusive.
- Important in many applications: political campaigns, micro-targeting on social media, rating systems.

Information spillovers as novel ingredient in info-design problem.

Challenges:

- How to model information spillovers and implied constraints?

This Paper

Information Spillovers:

- Social ties modeled as *directed network*.
- **Baseline assumption**: information flows mechanically in the network (non-strategic).
- Later: enrich information spillovers with strategic considerations.

We study two cases:

- Unconstrained designer: she can target each player **directly**.
- Constrained designer: she can target only a subset.

Main Results

A **toolbox** for characterizing and solving these problems:

1. Characterize feasible outcomes in terms of obedient recommendations.
   - Recommendations robust to information spillovers.
   - System of linear inequalities.

2. Effects of "deeper" networks on outcomes, payoffs and information.

3. Bounds on designer's payoff robust to wide range of communication forms (strategic and not).

4. A tractable way to solve constrained problems.

Information Design and Persuasion: Kamenica and Gentzkow (2011), Bergemann and Morris (2016, 2018), Mathevet, Perego, and Taneva (2017), Best and Quigley (2017), Galperti (2018), Inostroza and Pavan (2107).

Optimal Targeting. Grannoveter (1978), Banarjee et al. (2013), Jackson and Storm (2017), Akbarpour et al. (2017), Morris (2000), Sadler (2018).

Social/Observational Learning. Banerjee (1992), Bikhchandani et al. (1992), Smith and Sorensen (2000), Acemoglu et al. (2011), Golub and Sadler (2017) model

Timeline

1. Information-provision phase.

   Designer provides information to players

2. Communication phase:

   Information spillovers governed by network structure.

3. Game phase:

   Players interact in game using collected information.

Primitives

- Finite set of players N.
- Finite set of states Ω; prior belief μ.
- **Communication Network**: players are connected on **directed** network $E \subseteq N^2$.
- A **path** $j \to i$: a sequence $(i_1, \ldots, i_m) \subseteq N$ s.t. $i_1 = j$, $i_m = i$, and $(i_k, i_{k+1}) \in E$ for all $k = 1, \ldots, m-1$.

Information and Spillovers

- An information structure (S, π) is a map $\pi : \Omega \to \Delta(S)$, with $S := \times_i S_i$ finite. Denote Π set of all info structures.
- **Baseline assumption.**
  - If $j \to i$, player i learns signal s_j.
  - Information spillovers governed by communication network.
- Later: richer communication model with strategic considerations.
- **Remark:** Communication network E induces map

$$f_E : \Pi \to \Pi, \qquad f_E(\pi) = \pi'$$

Game

- After communication stage, players interact in game.
- A_i is finite action space of player i
- EU payoff: $u_i : \Delta(A) \times \Omega \to \mathbb{R}$, where $A := \times_i A_i$.
- Basic game is $G := (\Omega, \mu, (A_i, u_i)_{i \in N})$.
- Solution concept: $\text{BNE}(G, \pi)$

Information Design

- Designer's payoff $v : \Omega \times A \to \mathbb{R}$, common prior μ.
- Pick information structure π
  - Unconstrained: $\pi \in \Pi$.
  - Constrained: $\pi \in \Pi_C \subsetneq \Pi$, (e.g., optimal targeting).
- Let $V(\pi) := \max_{\sigma \in \text{BNE}(G,\pi)} \sum_{a,s,\omega} v(a,\omega)\sigma(a|s)\pi(s|\omega)\mu(\omega)$.
- Information-design problem: $V_E^* = \sup_{\pi \in \Pi_C} V(f_E(\pi))$

discussion

Our baseline communication model is stark, but simple:

- Whenever a link exists, information *will* flow.

Important baseline for two reasons:

1. Simplicity highlights qualitative implications of info spillovers.

2. **Result:** We show it is **worst-case** scenario for designer across wide range of communication processes (strategic and not).

unconstrained designer

Objective: provide simple characterization of **feasible** outcomes given communication network E.

Example

$\Omega := \{L, R\}$, prior $\mu(R) = \frac{1}{3}$.

Two players, $A_i = \Omega$, same preferences:

$$u_i(\omega, a_1, a_2) = \begin{cases} 1 & \text{if } a_i = \omega, \\ 0 & \text{else} \end{cases}$$

Designer's objective:

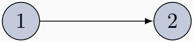
$$v(\omega, a_1, a_2) = v(a_1, a_2) = \begin{cases} 1 & \text{if } (a_1, a_2) = (\omega, L), \\ 0 & \text{else} \end{cases}$$

Example

Suppose $E = \emptyset$:

- Solution: $\pi(s_1 = \omega, s_2 = L \mid \omega) = 1$ for all $\omega \in \Omega$.
- Outcome: players' actions are independent.

Now suppose $E \neq \emptyset$:



- For all $\pi \in \Pi$, player 2 more informed than player 1.
- No longer feasible for designer to induce outcome above.

Objective: characterize feasible outcomes given E.

Notation

- Let $F_i := \{j \in N : i \to j\}$ be the set of *followers* of player i.
- Let $R_i := \{j \in N : j \to i\}$ be the set of *sources* for player i.

If (S, π) is initial information structure:

- s_{R_i} vector of signals player i learns.
- s_{-R_i} vector of signals player i does not learn.

Outcome function maps states into players' behavior

$$x : \Omega \to \Delta(Z) \qquad \text{with } Z := Z_1 \times \ldots \times Z_n$$

and $x(\cdot|\omega)$ has finite support for all ω.

Interpretation: Designer *recommends* each player *how to play* in G.

- Usually, recommendations defined as pure actions: $Z_i = A_i$.
- More generally, recommendation is a "way to play the game," i.e., pure or *mixed* action: $Z_i := \Delta(A_i)$.

When $E \neq \emptyset$, this generalization becomes necessary

Definition (Feasible Outcomes)

Outcome function x is **feasible** if there exists π and $\sigma \in \text{BNE}(G, f_E(\pi))$ such that

$$x(\alpha_1, \ldots, \alpha_N|\omega) = \sum_{s \in S} \pi(s|\omega) \prod_{i \in N} \mathbb{I}\{\sigma_i(s_{R_i})) = \alpha_i\}$$

Let $X(G, E)$ be the set of feasible outcomes for (G, E).

Definition (Obedience)

Outcome function x is **obedient** for (G, E) if, for all $i \in N$ and α_{R_i},

$$\sum_{\omega, \alpha_{-R_i}} \Big(u_i(\alpha_i, \alpha_{-i}; \omega) - u_i(a_i', \alpha_{-i}; \omega) \Big) x(\alpha_i, \alpha_{-i} | \omega) \mu(\omega) \geq 0, \quad a_i' \in A_i.$$

Characterization of Feasible Outcomes

The following result characterizes all feasible outcomes for any finite game G and communication network E.

Theorem

Fix G and E. Outcome function x is feasible *if and only if* it is obedient for (G, E).

- Basic trade-off: influencing one player's belief/behavior vs altering incentives of his followers.
- Feasible outcomes given by linear inequalities $\Rightarrow$ linear program.
- Simpler than dealing with information structures, spillovers, and equilibrium strategies.

Extreme Cases

To gain intuition, two extreme cases:

- Empty network, $E = \emptyset$.
- Complete network, $E = N^2$.

Extreme Cases: Empty Network

Information cannot flow $\Rightarrow$ standard information-design problem.

- Note: $E = \emptyset \Rightarrow R_i = \{i\}$.
- Obedience reduces to

$$\sum_{\omega,\alpha_{-i}} \Big(u_i(\alpha_i, \alpha_{-i}; \omega) - u_i(a_i', \alpha_{-i}; \omega) \Big) x(\alpha_i, \alpha_{-i} | \omega) \mu(\omega) \geq 0,$$

  for all i, α_i, and $a_i' \in A_i$.
- This condition is **equivalent** to obedience for Bayes Correlated Equilibria (Bergemann and Morris (2016)).

Extreme Cases: Complete Network

Complete network: As if players publicly *announced* private signals.

- Note: $E = N^2 \Rightarrow R_i = N$.
- Obedience reduces to:

$$\sum_{\omega} \Big(u_i(\alpha_i, \alpha_{-i}; \omega) - u_i(a_i', \alpha_{-i}; \omega) \Big) x(\alpha_i, \alpha_{-i} | \omega) \mu(\omega) \geq 0, \quad a_i' \in A_i.$$

Discussion

Between two extreme cases:

$$\emptyset \subseteq E \subseteq N^2$$

- Rich constraints on what is feasible.
- Network approach permits to govern complexity in simple and tractable way.

Proof

Intuition behind proof:

- *If.*
  - Obedient x can be seen as info structure.
  - Trivial strategies.
  - Even conditional on what players learn, obedience implies strategies are a BNE.

- *Only if.*
  - x feasible implies existence of π and σ.
  - Every i learns $s_{R_i} \Rightarrow$ learns sources' **mixed** behavior via σ.
  - $\sigma(s_{R_i})$ best response to σ_{-i}, knowing $\sigma(s_{R_j})$ for all $j \in R_i$.
  - Leading to obedience.

Why do we have to generalize the notion of recommendation to $\Delta(A_i)$?

Matching pennies:

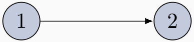
		P2	
		H	T
$P1$	H	$1, 0$	$0, 1$
	T	$0, 1$	$1, 0$

- Complete information, unique equilibrium (fully mixed). No scope for designer.

Suppose $E = \emptyset$. The only *feasible outcome* function x is

		P2	
		H	T
$P1$	H	$\frac{1}{4}$	$\frac{1}{4}$
	T	$\frac{1}{4}$	$\frac{1}{4}$

Now suppose $E \neq \emptyset$:



- Previous x no longer obedient.
- **No** outcome function in pure strategies can be obedient.
- Failure to represent reasonable outcome via recommendations.
- Simple generalization of the notion of *recommendation*.

		$P2$
		$\alpha_2 = (\frac{1}{2}, \frac{1}{2})$
$P1$	$\alpha_1 = (\frac{1}{2}, \frac{1}{2})$	1

comparative statics

Depth of Communications

How do changes in communication network E affect:

- Feasible outcomes?
- What information the designer provides?

A simple order on communication networks:

Definition

E' is *deeper* than E if, for all $i \in N$, player i's followers in E are also followers in E' (i.e., $F_i' \supseteq F_i$).

Proposition

$X(G, E') \subseteq X(G, E)$ for all G *if and only if* E' deeper than E.

- Deeper networks limit designer's ability to keep "local" information from spreading.
- Designer can "replicate" information spillovers, but cannot "undo" information spillovers ($\neq$ money).
- *Only if* part: *network depth* is the "right" order on networks.

Do players become more informed as network gets deeper?

- Rank info structures by informativeness in multi-player context.
- π is **more informative** than π' for player i if i's signals from π dominate i's signals from π' in Blackwell's sense.

Definition

E' **aggregates more** information than E if, for all $\pi \in \Pi$ and $i \in N$, $f_{E'}(\pi) \in \Pi$ is *more informative* than $f_E(\pi) \in \Pi$.

Proposition

E' aggregates more info than E *if and only if* E' is deeper than E.

- In socia-learning literature, information aggregation viewed as desirable property. (e.g., herding)
- Networks aggregate more information $\Rightarrow$ better social outcomes.
- Distinction with our framework: π is endogenous & arbitrary.
- **Common wisdom overturned**: networks that aggregate more information can lead to Pareto inferior outcomes.
  - More aggregation weakens third party's incentive to provide information.

Summary: as communication network becomes deeper, it

1. shrinks set of feasible outcomes.
2. decreases scope for benefiting from belief meddling.
3. makes players more informed (keeping π fixed).

richer communication

Overview

- Baseline assumption: very simple form of communication.
- Now consider richer forms of communication (strategic and not).
- Baseline model is a special case and provides **bounds** on designer's payoff for broad class of communication forms.

A Richer Communication Model

- K rounds of communication.
- At each round, player i sends (possibly different) messages to **her neighbors** in E.
- **Assumption**: finite K and message spaces, but sufficiently rich to impose no physical restrictions on communication.
- Player i's communication strategy: map ξ_i from histories of received/sent messages to new messages to be sent.

A Richer Communication Model

Profile ξ can represent different things:

- Truthful belief announcement.
  - A common model in diffusion games.
  - Micro-foundation of our baseline communication model.
- Observational learning (Golub and Sadler (2017)).
- Strategic communication (cheap talk, verifiable messages, etc).

We allow for a broad class of underlying models for ξ, but assume ξ is well defined for every initial π.

Remark

Fix E. Every profile of communication strategies ξ induces a map

$$f_{\xi,E} : \Pi \to \Pi.$$

Denote $V^{\star}_{\xi,E} := \sup_{\pi} V(f_{\xi,E}(\pi))$.

Payoff Bounds

Theorem (Payoff Bounds)

Fix basic game G and network E. Let ξ be any profile of communication strategies. Then,

$$V^{\star}_{\emptyset} \geq V^{\star}_{\xi,E} \geq V^{\star}_{E}$$

- Baseline model bounds designer's payoff, irrespective of details of communication form.
- Computing $V^{\star}_{\xi,E}$ can become easily unfeasible, especially in large networks and for strategic communication.
- Our bounds can be computed with linear programming.

constrained designer

Optimal Targeting

- With large network, unconstrained designer may be unrealistic.
- More plausibly, designer can **target** a small subset, **exploiting** social connections to spread her messages.
  - Information spillovers can now help the designer.
- Bridge literatures on *information design* and *optimal targeting/seeding*.
  - Novel dimension of belief manipulation: what information to convey, in addition to which players to target.

Theoretical viewpoint:

- Standard info design: **private** and **direct** information provision.
- **Before:** relaxed **privacy** and analyzed implications.
- **Now:** relax **direct** provision and analyze implications.

- Suppose designer targets at most $m < N$ players.
- Let $M \subset N$, with $|M| = m$, be the set of targets.
- Constrained target $\Leftrightarrow$ constrained info structures:

$$\Pi_M := \Big\{(S, \pi) \in \Pi : |S_i| = 1, \ \forall\, i \notin M\Big\}$$

- Designer value is given by: $V_E^\star(M) := \sup_{\pi \in \Pi_M} V(f_E(\pi))$.
- Optimal targeting problem:

$$\max \Big\{V_E^\star(M) \mid \text{s.t. } M \subseteq N \text{ and } |M| = m\Big\}$$

- Back to baseline assumption on information spillovers.
- **Goal:** characterization of feasible outcomes given M and E.

Definition

Outcome function x is M-**feasible** if there exists $\pi \in \Pi_M$ and $\sigma \in \mathrm{BNE}(G, f_E(\pi))$ such that

$$x(\alpha_1, \ldots, \alpha_N | \omega) = \sum_{s \in S} \pi(s|\omega) \prod_{i \in N} \mathbb{I}\{\sigma_i(s_{R_i})) = \alpha_i\}$$

for all $\alpha \in Z$.

Definition (M-Obedience)

Outcome function $x : \Omega \to \Delta(Z)$ is **M-obedient** for (G, E) if there exists $\kappa : \Omega \times Z \to \Delta(B)$ such that

1. $B := \times_{i \in N} B_i$, where B_i is finite and $|B_j| = 1$ for $j \notin M$.
2. For every i, every b_{R_i} fully reveals the recommended α_i.
3. For every i, α_i, b_{R_i} and $a_i' \in A_i$,

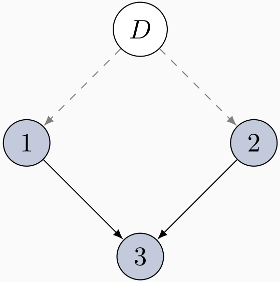
$$\sum_{\omega, \alpha_{-i}, b_{-R_i}} \Big(u_i(\alpha_i, \alpha_{-i}, \omega) - u_i(a_i', \alpha_{-i}, \omega) \Big) \kappa(b_{R_i}, b_{-R_i} | \alpha, \omega) x(\alpha_i, \alpha_{-i} | \omega) \mu(\omega) \geq 0$$

Call 2. “invertibility”

M-Obedience

M-obedience requires extra tool, κ.

- *Targeted* player $\sim$ designer's intermediary to *non-targeted* players.
- Interpretation: if $i \in M$, the realization of b_i contains:
  - Recommendation for i.
  - Parts of the recommendations for i's followers.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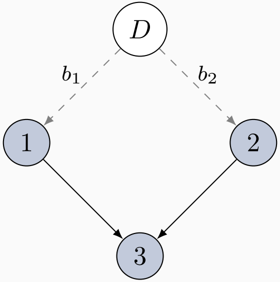

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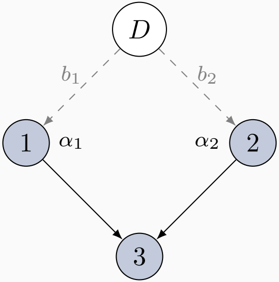

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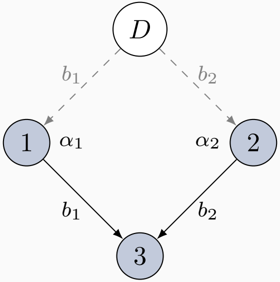

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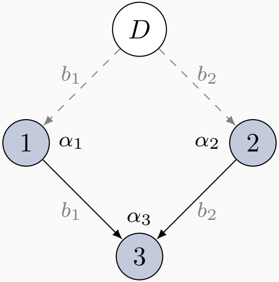


Theorem

Fix game G, network E, and targets M. Outcome function x is M-feasible *if and only if* it is M-obedient.

Remarks

- M-obedience $\Rightarrow$ Obedience: $X(G, E, M) \subseteq X(G, E)$

Extreme cases:

- If $M = N$, M-obedience is equivalent to obedience.
- If $M = \emptyset$, $X(G, E, \emptyset) = BNE(G, \mu)$

This demonstrates that

- $X(G, E, M)$ can fail to be convex, unlike $X(G, E)$.

Remarks

- Theorem maintains approach of unconstrained problem:

  Information-design problem as behavior recommendations.

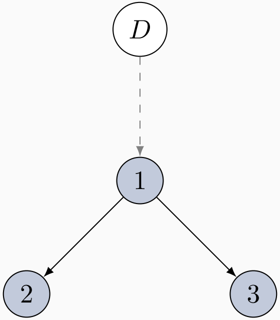
- This helps comparison and illustrates new challenges.

- Indirect communication requires richer language $\Rightarrow b \neq \alpha$.

  Use targets to reach non-targets with right message.

An Equivalence Result

- Yet, M-obedience is not a trivial requirement.
- Complexity stems from distinctive feature of constrained problem:
  - Unconstrained: i learns about his sources.
  - Constrained: i learns about his sources *and his followers*;
    he is used as **information intermediary**.
- For some cases, drastic simplification: e.g., $|M| = 1$.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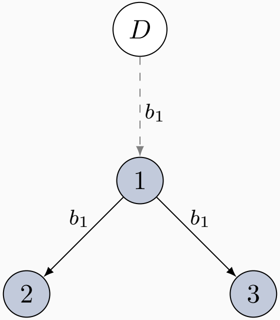
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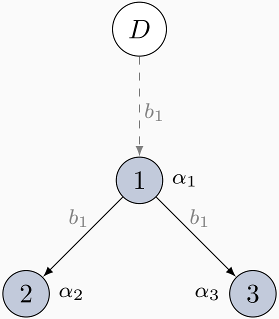
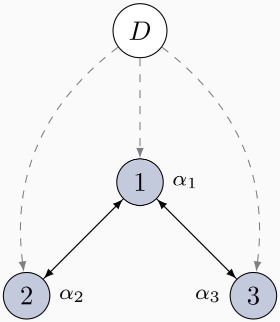
An Equivalence Result



An Equivalence Result



An Equivalence Result

Fix game G and network E.

Suppose designer can target a single player ($|M| = 1$).

- M-constrained problem is equivalent to an *unconstrained* problem where all links **to** followers of targeted i are made bi-directional.
- M-Obedience $\Leftrightarrow$ Obedience.
- Solve optimal targeting with toolbox for unconstrained problem: linear programming.

summary

Summary

We study optimal design problem with information spillovers under direct and indirect provision.

- Characterize feasible outcomes under baseline assumption.
- Derive payoff bounds for wide range of communication models (strategic and not).
- Simple method to solve unconstrained and constrained cases.

Summary

In the works:

- Characterization of *optimal* outcomes via linear programming
  Duality approach offers qualitative insights generalizing Bayesian persuasion and beyond.
- Exogenously informed players.
- Designer uncertain about network structure (network modeled as players' exogenous information)

Thank You!