Credibility and Flexibility in Communication

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Abstract

We study sender-receiver games of verifiable disclosure in which the sender's evidence is a noisy signal of her private information. Our paper explores a tradeoff created by verifiability. By disciplining the sender's claims, verifiability makes her more credible, which can improve her ability to transmit information. At the same time, by restricting how flexibly she can use language, it can impede her ability to communicate her private information. We show that unraveling is sustainable in equilibrium if and only if the sender's bias is sufficiently large. When preferences are more aligned, equilibria exist in which the sender uses silence in a flexible, type-dependent way to transmit more information than is contained in the verifiable evidence she possesses. Comparing verifiable disclosure with cheap talk, we show that flexibility is more valuable under greater alignment, whereas credibility is more valuable under greater misalignment. We also show that more informative evidence need not improve equilibrium communication, and that even uninformative evidence can sustain disclosure outcomes more informative than cheap talk.

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1 Introduction

A central lesson of the classic disclosure literature, beginning with Grossman (1981) and Milgrom (1981), is that verifiability is a positive force in communication.[1] In many sender-receiver environments, when the sender can back her claims with hard evidence, the "unraveling" logic leads her to disclose all the evidence, including evidence that is a priori unfavorable. As a result, verifiability mitigates, and often eliminates, the frictions created by asymmetric information. This paper qualifies that lesson. We show that when the sender's private information is richer than what her evidence can certify, verifiability creates a fundamental tradeoff between *credibility* and *flexibility*. The constraints imposed by verifiability discipline what the sender can say and under which contingencies, thereby enhancing her credibility, but they also limit how flexibly she can use the available language, which may hinder information transmission. Whether verifiability helps communication thus depends on the balance between these two forces, which we explore in this paper.

We study a class of sender-receiver games in which the sender communicates with the receiver by disclosing verifiable evidence. We depart from the standard disclosure setting in two ways. First, the sender knows more than the evidence can prove. In our model, the sender privately observes a payoff-relevant state, but the evidence she can disclose is only a noisy signal of that state. For instance, an executive knows more about the prospects of a firm than the accounting numbers reveal, a doctor may know more than a patient's test results reveals, and a job applicant knows more about her qualifications than what appears on her resume. Thus, disclosing evidence does not exhaust what the sender knows. Second, sender's and receiver's preferences are only partially misaligned. A bias parameter, as in Crawford and Sobel (1982), governs the degree of misalignment: while the sender would like the receiver to choose a higher action than the receiver's ideal one, both parties value decisions that are tailored to the state.

These two ingredients—noisy evidence and partially aligned preferences—change the economics of verifiable disclosure. In the standard model—where the sender can verifiably disclose her private information and her payoff is increasing in the receiver's action—verifiability is valuable because, by imposing state-dependent restrictions on the sender's strategy, it helps support full disclosure as an equilibrium. In our setting, verifiability still disciplines the sender,

[1] For reviews of this literature, see Verrecchia (2001), Milgrom (2008), Dranove and Jin (2010), and Beyer et al. (2010).

but the resulting language restrictions can now hinder communication. Because the sender knows more than her evidence can prove, she may want to use the same piece of evidence differently in different states, or use silence in an informative manner. This creates a tradeoff absent from the classic benchmark: a tension between credibility and flexibility.

To fix ideas, consider the benchmark outcome of *full evidence disclosure* (FED), in which the sender always discloses her evidence (that is, the realized signal) and the receiver therefore learns the full verifiable content of the sender's information. In the classic literature, this outcome is supported in equilibrium and is efficient, in the sense that it is the receiver's preferred outcome. Our first result shows that full-evidence disclosure can be supported in equilibrium if and only if the sender's bias is sufficiently large. Hence, when sender and receiver are sufficiently aligned, there is no equilibrium that induces the FED outcome. Moreover, in that case there exist equilibria that are strictly more informative than the FED outcome. Thus, full disclosure is not only infeasible but also inefficient. The reason is that in our setting silence is not merely a way to conceal unfavorable evidence. Because the sender knows more than the evidence can prove, silence can be used productively as part of the equilibrium language. Thus, verifiable communication is more flexible than the standard disclosure literature suggests.

This result sharply distinguishes our setting from the classic disclosure literature. In standard models, failures of unraveling arise because senders have incentives to pool, by concealing unfavorable evidence.[2] In our environment, full disclosure can fail for the opposite reason: the sender may want to separate more finely than is possible under the FED outcome. This is because he sender's private information is richer than the evidence alone. Verifiable communication therefore need not collapse to the information contained in disclosed evidence; it can also exploit the sender's discretion over when to remain silent.

Our second result is that this flexibility of verifiable communication remains limited and these limits are consequential. We demonstrate these limits by comparing the outcomes of communication when we remove the verifiability constraints thereby entering the world of cheap talk. In verifiable communication, the sender can disclose a realized signal or stay silent, but she cannot freely choose among all messages in all contingencies. This constraint is harmful when sender and receiver are closely aligned. In that case, the main obstacle to communication is not lack of credibility but lack of expressive power. We show that, when preferences are

[2]For example, see Dye (1985) or Jovanovic (1982).

sufficiently aligned, cheap talk can outperform verifiable disclosure because it gives the sender a more flexible language. When preferences are sufficiently misaligned, the opposite holds because the discipline imposed by verifiability becomes more important.

To understand the mechanics of this tradeoff more deeply, we ask which of two channels drives the value of verifiability: the *constraint channel* (verifiability limits imitation and thereby enhances credibility) or the *information channel* (disclosed evidence is statistically informative about the state). We disentangle these channels by studying a benchmark in which the evidence structure is completely uninformative—the distribution of signals is independent of the state.

To isolate the first channel, we study a benchmark in which evidence is completely uninformative. Even though evidence carries no direct information about the state, verifiable communication can be strictly informative—and can even outperform cheap talk. The mechanism is that verifiable evidence, though uninformative, is stochastically available only to some sender types. This exogenous source of randomization limits the sender's ability to mimic across types and can sustain informative separation that would not be credible under cheap talk. Thus, the intrinsic informativeness of evidence is not necessary for verifiability to improve communication; what matters is the constraint that verifiability imposes on feasible messages.

To evaluate the second channel, we then ask whether making evidence more informative—in the Blackwell sense—systematically improves equilibrium communication. A natural conjecture is that it should. We show that this conjecture is correct at the extremes: when the bias is zero (common interests) or very large (type-independent preferences), Blackwell-more-informative evidence leads to more informative equilibrium outcomes. But for intermediate values of the bias, the conjecture fails. More informative evidence can make both sender and receiver worse off. The reason is that equilibrium communication depends not only on how informative disclosed evidence is, but also on how the induced feasibility constraints shape the sender's ability to use the available language.

A natural application of our framework concerns internal organizational reporting and performance evaluation. In many firms, lower-level managers and employees know more than formal metrics can capture about project quality, execution problems, customer relationships, and future prospects. Reporting systems therefore face the same tradeoff as in our model. Flexible, informal, and partly unverifiable communication—such as narrative updates, contextual explanations, and subjective evaluations—can convey information that standardized metrics miss.

When interests are relatively aligned, a rigid reliance on standardized performance indicators can therefore suppress useful information: organizations may learn more when they allow some discretion in exception reporting, escalation, and narrative contextualization of hard measures. This interpretation is closest in spirit to Baker et al. (1994), who study the joint use of objective and subjective performance measures when formal measures are imperfect, and it is reinforced by the evidence in Gibbs et al. (2004) that subjectivity is used more when quantitative measures are weak, long-term intangibles and organizational interdependencies are important, and relational trust is stronger. It is also consistent with Keating (1997), who shows that firms vary divisional performance metrics with growth opportunities and cross-divisional effects, suggesting that performance-evaluation systems are tailored to the limits of standardized measures. By contrast, when agency problems are more severe, organizations have stronger incentives to rely on formalized reporting systems—standardized scorecards, audit trails, and approval protocols—because these devices constrain strategic communication even at the cost of suppressing nuance. This margin is closely related to Aghion and Tirole (1997), where the allocation of real authority depends on the information structure and the precision of performance measurement. More broadly, the management-control literature suggests that formal and informal controls should be viewed as interacting bundles whose effects are context dependent rather than uniformly substitutable or complementary (Kreutzer et al., 2016; Lumineau et al., 2023).

Our results also speak to the role of standardized grading and other legibility schemes in markets for differentiated goods. A classic example is the standardization of commodity grades, which, as documented by Cronon (1991), facilitated trade by replacing rich but nonportable local descriptions of quality with a small set of verifiable categories. In the language of our model, the key benefit of such schemes is not that they necessarily capture all relevant information, but that they convert flexible but difficult-to-verify claims into a language that is credible and interpretable even in arm's-length exchange. This makes standardization especially attractive when parties lack shared context, repeated interaction, or trust. At the same time, the move to standardized categories can suppress information about dimensions of quality that are observable to local traders but not encoded in the formal grading system. In that sense, our framework offers a communication-based interpretation of one margin of the broader legibility debate emphasized by Scott (1998): formal categories can improve communication by making claims more credible, but they can also reduce informativeness by displacing richer contextual knowledge. We view this as a narrow mechanism within a much broader historical process, not as a full account of the market-thickening and institution-building effects of standardization.[3]

A related, though necessarily looser, application concerns the formalization of customary land claims into legal title. In many customary tenure systems, rights are communicated through a context-dependent vocabulary of social relationships, overlapping use claims, and local practice rather than through a single standardized document. This is the sense in which what German (2022) calls "strategic ambiguity" may sometimes be informative rather than merely dysfunctional: within communities that share local knowledge, flexible and partly informal claims can convey nuances about access, seasonality, and use rights that rigid legal categories do not capture. Our framework suggests that formalization is most likely to improve communication in outsider-facing transactions—such as sales, disputes with distant parties, or borrowing against outside creditors—where credibility and portability of claims are paramount, as in the logic emphasized by de Soto (2000). By contrast, when claims are interpreted within ongoing local relationships, forcing communication into a rigid, verifiable language may reduce informativeness even if it increases formal clarity. We stress, however, that this is only a communication-based interpretation of one margin of legal formalization. It abstracts from the many other channels through which titling affects outcomes, including enforcement, collateral value, investment incentives, and political authority. For that reason, our framework is better viewed as helping to organize part of the discussion in Platteau (1996) and Fitzpatrick (2006) than as a general theory of tenure security or property-rights reform.

Related literature. *(Provisional and Incomplete)* Our paper contributes to several strands of the literature. First, it relates to the disclosure literature initiated by Grossman (1981) and Milgrom (1981), where unraveling toward full disclosure is the central benchmark. We study a setting in which disclosure does not exhaust the sender's information, and show that the full-disclosure benchmark can fail for novel reasons even though evidence is verifiable. Second, we connect to the cheap-talk literature following Crawford and Sobel (1982) by allowing partial preference alignment and comparing the expressive power of unverifiable and verifiable communication.

Our analysis of uninformative evidence is related to a broader idea in the communication literature: frictions in the communication technology can sometimes improve information trans-

[3] For broader discussions of the costs of standardization and commensuration, see Espeland and Stevens (1998) and Karpik (2010).

mission by limiting imitation or otherwise disciplining sender behavior. In Myerson (1991) carrier-pigeon example, and more generally in Blume et al. (2007), the relevant friction is ex post noise in transmission: after the sender chooses a message, the communication channel may distort delivery or prevent the message from arriving. In Blume and Board (2013), the friction is instead tied to imperfectly shared language: communication is constrained by language competence, and equilibrium messages may convey information not only about the payoff-relevant state but also about linguistic ability. Our uninformative-evidence benchmark is related in spirit to both strands of work, but the mechanism is distinct. The key friction in our setting is an ex ante disclosure constraint induced by verifiability. Even when evidence is completely uninformative about the state, the sender can communicate only through evidence she actually has, and that restriction alone can sustain informative communication.

2 Model

2.1 Main Ingredients and Equilibrium

In our model, a sender privately observes a state and sends a message to the receiver, who then chooses an action that affects both players' payoffs. The sender is partially biased, in that she seeks to induce an action that is higher than what the receiver would like. The sender communicates by deciding whether or not to disclose a noisy signal whose realization is imperfectly informative about the state. We will compare two polar versions of this model: one in which signal disclosure is verifiable and one in which it is not.

More formally, let $\theta \in \Theta \triangleq [0,1]$ be an unverifiable state drawn from a prior distribution F with full support on $[0,1]$. We assume that F is atomless and admits a continuous density f. Let (S,π) be an *evidence structure*, where $S = \{s_1,\ldots,s_N\}$ is a finite set of possible signal realizations, endowed with a total order $s_1 < \cdots < s_N$, and the map $\pi : \Theta \to \Delta(S)$ specifies the conditional distribution of signals given the state. We assume $N \geq 2$ and that (S,π) satisfies the monotone likelihood ratio property (MLRP): for any $s_i > s_j$, the likelihood ratio $\pi(s_i \mid \theta)/\pi(s_j \mid \theta)$ is nondecreasing in θ. Throughout the paper, we also assume full support of the signal distribution: $\pi(s \mid \theta) > 0$ for all (θ, s). In other words, every signal can arise from every state.

We say that the evidence structure is *uninformative* if $\pi(s_i|\theta) = p_i$ for all θ, where $p_i > 0$

and $\sum_{i=1}^{N} p_i = 1$. That is, the signal distribution is independent of θ. Conversely, we say that the evidence structure is informative if it is not uninformative.

The sender privately observes (θ, s), her *type*. We will often refer to the realized signal s as the sender's *evidence*. She then sends a message m to the receiver. The message space is $M \triangleq S \cup \{\circ\}$, where $\circ$ is a shorthand symbol we interpret as "silence." After observing m, the receiver chooses an action $a \in A \triangleq [0,1]$. Preferences are quadratic. The receiver's payoff is $u(a,\theta) = -(a-\theta)^2$, and the sender's payoff is $v(a,\theta) = -(a-\theta-b)^2$, where $b \geq 0$ is an additive bias capturing preference misalignment.

We compare two polar communication technologies. Evidence is *verifiable* if a type-(θ, s) sender can either disclose s or remain silent; that is, $m \in M^V(\theta, s) \triangleq \{s, \circ\}$. Evidence is *unverifiable* if a type-(θ, s) sender can send any message in M; that is, $m \in M^U(\theta, s) \triangleq M$. We will sometime refer to the game with verifiable evidence as the *disclosure game* and to the game with unverifiable evidence as the *cheap-talk game*.

A sender's strategy is a map $\sigma : \Theta \times S \to \Delta(M)$. When evidence is verifiable, the sender's strategy must satisfy the verifiability constraint introduced above: for all (θ, s), $\sigma(m \mid \theta, s) = 0$ whenever $m \notin \{s, \circ\}$. In the game with unverifiable evidence, the sender's strategy is unrestricted. A receiver's strategy is a map $\alpha : M \to A$. Without loss of generality, we restrict attention to pure receiver strategies, since the receiver's payoff is strictly concave in a. A belief system is a map $\mu : M \to \Delta(\Theta \times S)$. The triple (σ, α, μ) is an assessment. The solution concept is Perfect Bayesian Equilibrium (PBE), which we often simply call an equilibrium. The definition of a PBE in our setting is standard and is therefore relegated to Appendix A.

2.2 Informativeness and Social Welfare

An *outcome* of the game is a joint distribution over the state and the action, $\lambda \in \Delta(\Theta \times A)$. Given an outcome λ, let $\mathbb{E}_\lambda[u(\theta, a)]$ and $\mathbb{E}_\lambda[v(\theta, a)]$ denote the receiver's and sender's expected payoffs, respectively. Let $W(\lambda)$ denote their sum, which we refer to as *social welfare*.

An outcome is *uninformative* if the receiver's action is constant in the state. It is said to be *informative* otherwise.

Definition 1. *Let λ and λ' be two outcomes. We say that λ is* **more informative** *than λ' if the social welfare under λ is weakly higher than under λ'.*

When λ is induced by an equilibrium, the sender's and receiver's expected payoffs are linearly related: $\mathbb{E}_\lambda[v(\theta, a)] = \mathbb{E}_\lambda[u(\theta, a)] - b^2$. It follows that, among equilibrium outcomes, any outcome that maximizes the receiver's ex ante payoff also maximizes the sender's ex ante payoff, and hence maximizes social welfare.

Next, we define a benchmark that will be used extensively.

Definition 2. *Consider the sender strategy $\sigma(m \mid \theta, s) = \mathbb{1}\{m = s\}$ for all (θ, s). Let α satisfy $\alpha(s) = \mathbb{E}[\theta \mid s]$ for every s. The* **Full Evidence Disclosure (FED)** *outcome is the joint distribution of (θ, a) induced by the strategy profile (σ, α). An equilibrium is a FED equilibrium if it induces the FED outcome.*

Note that, in many real-world settings, senders are required by law to disclose their verifiable evidence—for example, restaurants may be required to display hygiene cards. In our model, such a policy achieves the FED outcome. Moreover, whenever a FED equilibrium exists, the same outcome can be sustained without any external mandate: mandated disclosure is then self-enforcing in equilibrium.

Our game admits multiple equilibria. We call an equilibrium efficient if it induces an outcome that is most informative among all equilibrium outcomes, and hence maximizes social welfare. Formally, let Λ^{PBE} denote the set of equilibrium outcomes, and define $W^* \triangleq \sup_{\lambda \in \Lambda^{\text{PBE}}} W(\lambda)$, the efficient equilibrium social welfare. An equilibrium is efficient if it induces an outcome $\lambda^* \in \Lambda^{\text{PBE}}$ such that $W(\lambda^*) = W^*$. In general, an efficient equilibrium need not exist, because Λ^{PBE} need not be compact (and the supremum above need not be attained). Even when this is the case, it is still meaningful to study the efficient social-welfare level W^*, since it can be approximated arbitrarily well in equilibrium. That said, none of our results hinge on this technicality.

2.3 Discussion

We briefly pause to highlight some aspects of our model and to relate it to existing communication frameworks.

The game with unverifiable evidence is a fairly standard cheap-talk game, with the twist that the message alphabet is exogenously restricted: in any equilibrium, the sender can use at most $N + 1$ distinct messages (the cardinality of M). In this case, the evidence structure (S, π)—

despite being "objectively" informative—does not shape the equilibrium set beyond pinning down the size of the message alphabet.[4]

The most novel version of our model is the game with verifiable evidence. It combines two ingredients that are relatively understudied in the disclosure literature. First, the sender's and receiver's preferences are only partially aligned, as captured by the bias parameter $b \geq 0$. Such partial alignment is standard in the cheap-talk literature, but is less common in the disclosure literature (with the notable exception of Seidmann and Winter (1997)), where the sender is often assumed to have type-independent preferences. Second, evidence is a noisy signal about the state: every type (θ, s) assigns positive probability to every signal realization, so any sender can in principle disclose any piece of evidence. As a result, the sender cannot verifiably prove her type. This departs from canonical disclosure models, where it is typically assumed that a type-(θ, s) sender can send a message in $M^V(\theta', s')$ if and only if $(\theta', s') = (\theta, s)$.

Many real-world disclosure settings share the features of our model. For example, a student may disclose her standardized-test scores to a prospective employer, or a borrower may disclose her FICO credit score to a potential lender. In each case, the sender has private information about the payoff-relevant state—for instance, the student's skills or the borrower's creditworthiness—that goes beyond what her evidence can establish. That is, the disclosed evidence is verifiable yet only imperfectly informative about the underlying state.

From a technical perspective, the full-support assumption has important implications for the equilibrium set. When every signal can arise from every state, verifiability no longer disciplines off-path beliefs: any disclosed message is always feasible for any type. This removes a key force that pins down beliefs in canonical disclosure models, and it generates a fundamental source of equilibrium multiplicity in our setting. As a result, equilibrium analysis is considerably more delicate than in the standard case. Precisely because the equilibrium set is so rich, obtaining sharp results is nontrivial. Despite these challenges, we provide a substantive characterization of the equilibrium set.

A more minor assumption is that of quadratic payoffs. This is a common assumption in the cheap talk literature as it facilitates welfare discussion and calculations of equilibrium thresholds. We do not believe that the thrust of our results depends critically on this assumption.

[4]For a formal argument, see Appendix A.

3 The Disclosure Game

This section analyzes the disclosure game, that is, our communication setting with verifiable evidence. Section 3.1 establishes equilibrium existence and characterizes sender strategies in any equilibrium. Section 3.2 studies how the equilibrium set varies with the sender's bias.

3.1 Equilibrium Characterization

We begin by characterizing equilibrium outcomes, starting with existence.

Proposition 1. *Fix any $b \geq 0$. The disclosure game admits an uninformative equilibrium. Moreover, if evidence is informative, then the disclosure game also admits an informative equilibrium.*

The intuition behind the existence of an uninformative (babbling) equilibrium is simple but instructive. Consider a sender strategy that sends $m = \circ$ with probability one, so that the receiver chooses a constant action. Specify off-path beliefs so that, after any off-path message, the receiver takes the same action as after silence. This makes silence optimal for the sender, since any deviation induces the same action. Such off-path beliefs are admissible precisely because the sender privately observes θ and, under full support, every signal realization is feasible from every state. Hence, upon observing an off-path message $m = s$, the game imposes essentially no restriction on the receiver's beliefs. In particular, we can select beliefs that rationalize the same action as after silence.

Despite its simplicity, this observation highlights a key difference between our environment and standard disclosure models. In such models, babbling equilibria typically do not exist precisely because verifiability allow some types to profitably and credibly deviate away from babbling.

Proposition 1 also implies that, whenever evidence is informative, the disclosure game admits an informative equilibrium for every $b \geq 0$. This point may look innocuous at first, but it is not. A reader might be tempted to infer it from the existence of a FED equilibrium for all $b \geq 0$. As we will soon show, however, a FED equilibrium need not exist when b is small, so the existence of informative equilibria requires a separate argument.

Next, we show that the set of equilibrium outcomes can be fully characterized through a class of sender strategies that are "simple" in the following sense: they are pure (that is, they involve no randomization conditional on the sender's type) and are described by cutoff rules. The following definition introduces this class of strategies.

Definition 3. *A sender strategy $\sigma : \Theta \times S \to \Delta(M)$ is a* **generalized cutoff rule** *if there exist a partition (L, M, H) of the index set $\{1, \dots, N\}$ and cutoffs $(t_i)_{i \in L \cup H} \subseteq [0, 1]$ such that, for every type (θ, s_i),*

$$\sigma(m = s_i \mid \theta, s_i) = \begin{cases} 1 & \text{if } i \in L \text{ and } \theta \leq t_i, \\ 1 & \text{if } i \in H \text{ and } \theta \geq t_i, \\ 0 & \text{otherwise.} \end{cases}$$

A generalized cutoff rule partitions signals into three categories. Signals with indices in M are never disclosed by the sender. For signals with indices in L, the sender discloses s_i only when the state is sufficiently low, namely when $\theta \leq t_i$; otherwise she remains silent. Symmetrically, for signals with indices in H, the sender discloses s_i only when the state is sufficiently high, namely when $\theta \geq t_i$, and is silent otherwise. Thus, conditional on each signal realization, disclosure is governed by a single threshold in the state.

Lemma 1. *It is without loss of generality to restrict attention to equilibria in which the sender plays a generalized cutoff rule. More formally, every equilibrium of the disclosure game is outcome-equivalent to one in which the sender uses a generalized cutoff rule.*

This result implies that the set of equilibrium outcomes is unchanged if we restrict attention to generalized cutoff strategies. This technical result is useful for three reasons. First, it shows that mixed strategies are inessential in this game, which greatly simplifies the analysis. Second, it allows us to focus on strategies with a monotone partition structure. This echoes the classic monotone partitional characterization of equilibria in the cheap talk game (Crawford and Sobel, 1982). The difference here is that monotonicity operates separately for each realized signal and involves at most two regions, because, conditional on s_i, the sender effectively chooses between only two messages. Third, this result reassures the reader that the results in the next sections do not rely on pathological or strategically complex sender behavior.[5]

[5] The proof of Lemma 1 also yields an additional simplification. Whenever silence is on the equilibrium path, it is without loss of generality to focus on equilibria in which no on-path action coincides with the action taken after silence. This property will be convenient for several of our results, since it allows us to pin down the relevant cutoffs uniquely.

3.2 The Value of Silence

A benchmark insight in the disclosure literature—often called the unraveling principle—is that, under suitable conditions on preferences and the evidence structure, a sender who can verifiably disclose evidence will fully reveal it in equilibrium, even when the evidence is unfavorable. In such an equilibrium, disclosure eliminates the relevant information asymmetry between sender and receiver and thereby removes the inefficiencies that would arise under limited communication. In our setting, this benchmark corresponds to a FED equilibrium: the sender always discloses her realized signal, so the receiver learns the signal realization, which is the entire verifiable content of the sender's information.

Our first substantive result establishes that the existence of an equilibrium in which the evidence is fully disclosed critically depends on the sender's bias.

Proposition 2. *Suppose evidence is informative. There is a threshold* $\bar{b} \triangleq \frac{1}{2}\mathbb{E}[\theta \mid s_N]$ *on the sender's bias such that a FED equilibrium exists if and only if* $b \geq \bar{b}$*.*

The positive part of Proposition 2—the existence of a FED equilibrium for large b—is in line with the disclosure literature. This literature has overwhelmingly focused on the case in which the sender's preferences are type independent and shown that a FED equilibrium exists.[6] Following a similar logic, we show that when $b \geq \bar{b}$, a FED equilibrium can be supported by punishing deviations with off-path beliefs that are sufficiently pessimistic, for instance by specifying that the receiver's belief after silence induces action 0, the lowest action.

The more novel feature of Proposition 2 is its negative part: when the sender's and receiver's preferences are sufficiently aligned, there is no equilibrium in which the receiver learns the realized evidence with probability one. To build intuition, consider the extreme case of perfectly aligned preferences, $b = 0$, and suppose for contradiction that there exists a FED equilibrium in which the sender discloses her evidence with probability one. Let $a_\circ \in [0, 1]$ denote the receiver's action following the off-path message $m = \circ$. Consider a sender type $(\theta, s) = (a_\circ, s)$. This type can either send $m = \circ$, thereby inducing her first-best action $a_\circ = \theta$, or send $m = s$, thereby inducing the action $\mathbb{E}[\theta \mid s]$. Because evidence is informative, there exists some $\tilde{s} \in S$ such that $\mathbb{E}[\theta \mid \tilde{s}] \neq a_\circ$. Hence the type $(\theta, s) = (a_\circ, \tilde{s})$ strictly prefers sending $m = \circ$, a con-

[6]Note that, in our setting, if $b \geq 1$, all sender types prefer higher receiver actions. Therefore, we recover the standard case of type-independent sender preferences.

tradiction. In other words, there is no way of specifying of off-path beliefs (and thus no off-path action $a_\circ$) that can deter some type from deviating.

When $b < \bar{b}$, the sender has incentives to induce a richer set of equilibrium actions than the one induced by the FED outcome. In particular, a putative FED equilibrium would induce only N actions on path, whereas the communication technology can, in principle, support $N+1$. Some sender's types would then like to exploit this available flexibility to induce an additional action, thus enriching the equilibrium language. This "separating force" is behind the non-existence of FED equilibria for small b.

This force also clarifies why our result is distinct from earlier work in the disclosure literature on the failure of the unraveling principle (e.g., Dye (1985); Okuno-Fujiwara et al. (1990)). In that literature, the FED outcome cannot be supported in equilibrium for the opposite reason: the sender has incentives to coarsen the language associated with the putative FED outcome—for instance, because some sender types have incentives to imitate others in order to conceal unfavorable evidence. This "pooling force" induces fewer actions in equilibrium than would be supported under the FED outcome.

This contrast suggests that, in our setting, the sender may be able to transmit more information than is encoded in the FED outcome. More formally, it suggests that the efficient equilibrium outcome—that is, the equilibrium outcome that maximizes information transmission—may be strictly more informative than the FED outcome. The following result formalizes this intuition.

Proposition 3. *If $b \geq 1$, the FED outcome is efficient. Conversely,*

- *For all $b < 1$, there exists (F, S, π) under which the efficient equilibrium outcome is more informative than the FED outcome.*
- *For all (F, S, π), when $b = 0$, the efficient equilibrium outcome is more informative than the FED outcome. Under the disclosure regularity condition stated in Appendix B, this strict ranking persists for all sufficiently small b.*

Proposition 3 characterizes how the sender's bias affects whether the FED outcome is efficient. Let us unpack it.

The first clause is a faithful replication of the classic result in Milgrom (1981). When $b \geq 1$, the sender's preferences over the receiver's actions become type independent, that is, all types prefer higher actions. In this case, not only does a FED equilibrium exist, as established by Proposition 2, but it is also the most informative outcome that can be sustained in equilibrium. This leads to the familiar conclusion that policies mandating disclosure, that is, policies that force the sender to disclose verifiable evidence, are in theory ineffective: the efficient equilibrium already achieves the same outcome as such policies.

The rest of Proposition 3 establishes a partial converse. The key insight these results offer is that when the sender's and receiver's preferences are sufficiently aligned, there exist equilibria that are more informative than the FED outcome. We formalize this insight in two ways.

First, we show that for every $b < 1$ there exist a prior F and an evidence structure (S, π) such that the efficient equilibrium outcome is more informative than the FED outcome. We present an example in the next section, and therefore defer the discussion. Second, we establish a stronger statement: for every environment (F, S, π), there exists a bias level b small enough that the efficient equilibrium outcome is more informative than the FED outcome. To prove this result, we begin with the benchmark case $b = 0$, in which the sender and receiver have common interests. We study an auxiliary problem in which the sender can commit to a generalized cutoff strategy (see Definition 3) to maximize her expected payoff. This auxiliary problem abstracts from the sender's incentive constraints. We first show that the optimal commitment solution induces an outcome that is more informative than the FED outcome. The argument highlights that, when $b = 0$, the sender exploits the available language more flexibly than under FED: in particular, she induces $N + 1$ distinct on-path actions rather than only N. Finally, we show that this commitment outcome can be supported in equilibrium. The remainder of the proof uses a continuity argument to show that the same conclusion extends to all sufficiently small $b > 0$.

One practical implication of Proposition 3 is that, in settings in which the sender's bias is not extreme, mandated disclosure can be detrimental to information transmission. The sender can exploit her knowledge of the state, together with the flexibility (albeit limited) of verifiable language, to communicate more than what is encoded in the FED outcome. This is often (though not always) accomplished by putting all messages on the equilibrium path. In particular, silence can be used productively by the sender, which explains why allowing the sender discretion over whether to remain silent can improve communication.

We conclude by stressing that the results in this section hinge on the evidence structure we consider in this paper. Recall that evidence is noisy and that the sender observes θ. Thus, the sender typically knows more than what her verifiable evidence can prove. To see the implications of this assumption, consider a variant of the model in which the sender does not observe θ and instead observes only s. In that case, disclosure exhausts the sender's information about the payoff-relevant state: the sender can reveal all that she knows. The arguments behind Propositions 2 and 3 then break down. In particular, in this variant a FED equilibrium exists for all $b \geq 0$ and coincides with the efficient equilibrium outcome (see Proposition B.1 in Appendix B).

4 Flexibility and Credibility

The results from the previous section highlight the value—in terms of equilibrium informativeness—of allowing the sender to exploit the (limited) flexibility of a verifiable language. In our disclosure game, however, this flexibility is quite constrained. The sender effectively has only $N + 1$ messages, and N of them (the signal realizations) can be sent only in the corresponding contingencies, namely when that signal is actually realized. This naturally raises the question of what happens when the sender's flexibility is increased.

In this section, we contrast the disclosure model with its polar opposite: the game with unverifiable evidence (the "cheap-talk" game) introduced in Section 2. In this alternative communication environment, any sender type can send any message in $M = S \cup \{\circ\}$, with no verifiability constraints, that is, in a fully flexible manner.

The following result highlights the trade offs implied by the flexibility in the sender's communication abilities.

Proposition 4. *Suppose evidence is informative.*

- *When $b = 0$, the efficient equilibrium with unverifiable evidence is more informative than the efficient equilibrium with verifiable evidence. Under the regularity condition stated in Appendix B, this strict ranking persists for all sufficiently small b.*
- *When b is sufficiently large, the opposite holds.*

The result formalizes a simple insight, which is however novel in the literature to the best of our knowledge. The additional flexibility afforded by unverifiable evidence is a boon for communication when the sender's and receiver's preferences are sufficiently aligned. In particular, when $b = 0$ (the common-interest case), the efficient equilibrium of the cheap-talk game with $N + 1$ messages is more informative than the efficient equilibrium of the disclosure game. Although the sender also has $N + 1$ messages in the disclosure game, verifiability constraints restrict which messages can be sent in which contingencies, thereby limiting how flexibly the sender can use the available language and thus hindering information transmission.

In contrast, flexibility becomes a liability when the sender's preferences are sufficiently misaligned with those of the receiver. In those cases, the lack of flexibility can enhance the sender's credibility and thereby facilitate communication. To fix ideas, consider $b \geq \max\{\frac{1}{2}, \bar{b}\}$. When $b \geq \frac{1}{2}$, all equilibria of the cheap-talk game are uninformative (see Lemma B.5 in Appendix B). When $b \geq \bar{b}$, Proposition 2 establishes that the efficient equilibrium of the disclosure game is at least as informative as the FED outcome. The result then follows.

Finally, the results so far have practical implications. The verifiability of information—and, more broadly, institutions that promote verification in communication—is generally viewed as a positive force. The prevailing lesson from the disclosure literature is that verifiability facilitates information transmission. Our results qualify this view. In settings with realistic features—in particular, a sender whose bias is not necessarily extreme and a noisy evidence structure—verifiability may hinder rather than facilitate communication. The reason is that verifiability can restrict the sender's ability to choose the language that best conveys her private information.

4.1 An Example: The Case of Uninformative Evidence

Our results so far have highlighted a trade-off between flexibility and credibility. The reader may notice that our comparison between the disclosure and cheap-talk games conflates two distinct drivers of the differences we have documented. Relative to the cheap-talk game, the disclosure game differs in two respects. First, the sender's strategy is constrained: she can send message s only if she has in fact received signal s, and therefore lacks the flexibility available under cheap talk. Second, when evidence is informative, signals admit an "objective" interpretation: observing s is informative about θ because different values of θ generate s with different probabilities.

The combination of these forces—strategic constraints and the intrinsic informativeness of signals—underlies the qualitative results we have discussed. It is therefore natural to ask which

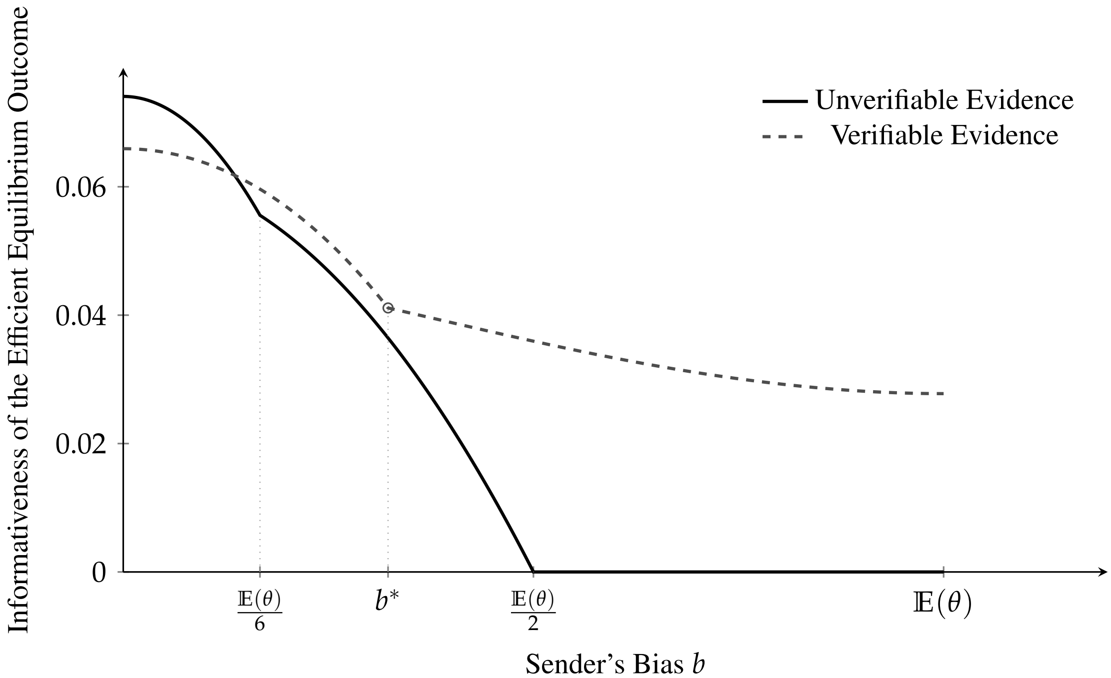

Figure 1: Efficient equilibrium outcomes with verifiable and unverifiable evidence: Linear evidence $\pi(s_1|\theta) = 1 - \theta$ and $\pi(s_2|\theta) = \theta$, uniform prior, $N = 2$. Under verifiable evidence, the efficient equilibrium has two cutoffs $0 < t_1 < t_2 < 1$ with disclosure only if $\theta \leq t_1$ given s_1 and $\theta \geq t_2$ given s_2 for $b < b^\star$; a single cutoff $0 < t_2 < 1$ with disclosure only if $\theta \geq t_2$ given s_2 for $b \in [b^\star, 1/2)$; and coincides with the FED equilibrium for $b \geq 1/2$.

component is doing the work: the constraints themselves, the informativeness of signals, or their interaction. This section addresses this question directly and shows that intrinsic signal informativeness is not essential for many of our results.

To disentangle the two forces, we consider an extreme benchmark in which the evidence structure (S, π) is uninformative. Recall that this means $\pi(s_i \mid \theta) = p_i \in (0, 1)$ for all θ, so the distribution of signals is independent of θ. With uninformative evidence, any information transmission in the disclosure game must come purely from the presence of verifiability constraints on the sender's strategy. We begin by presenting our results for the benchmark under a uniform prior F. This assumption yields a particularly clean characterization and provides a useful illustration of the forces at work. The qualitative takeaway, however, extends beyond the uniform case, as we establish later.

We have the following result, an example of which is depicted in Figure 2.

Proposition 5. *Let F be uniform and let (S, π) be uninformative. There exists $b^\circ \in (1/8, 1/4)$,*

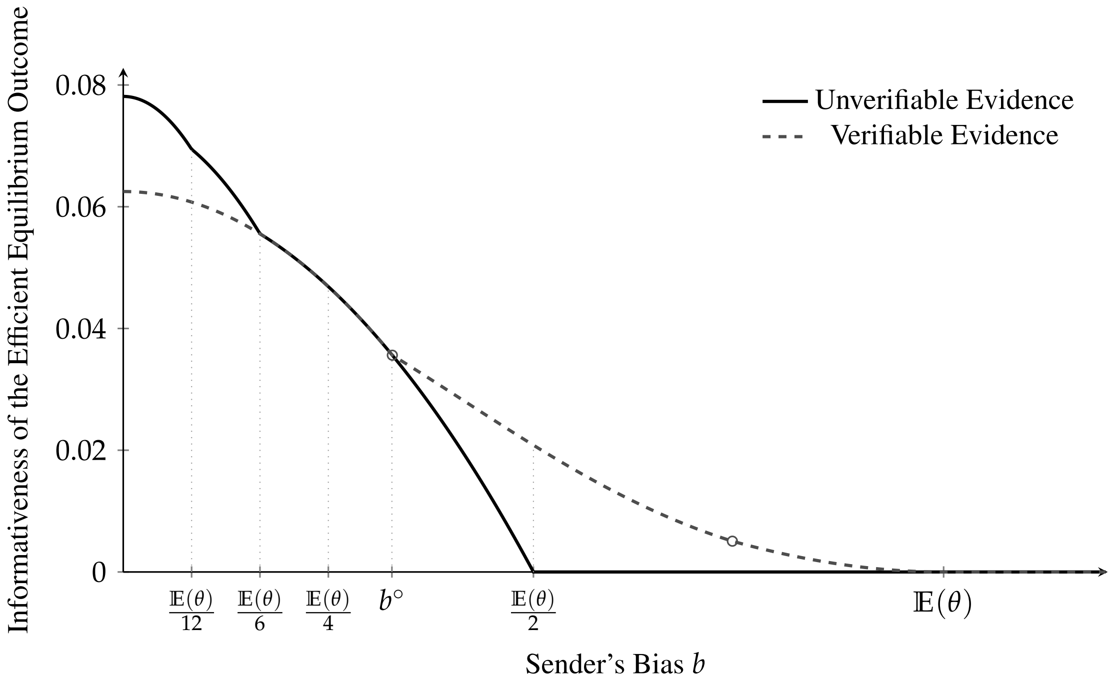

Figure 2: Efficient equilibrium outcomes with verifiable and unverifiable evidence: Uninformative evidence, uniform prior, $N = 3$.

which depends on π, such that:

- *If $b < \frac{\mathbb{E}(\theta)}{6}$, the efficient equilibrium outcome is more informative when evidence is unverifiable.*
- *If $b \in \left(\frac{\mathbb{E}(\theta)}{6}, b^\circ\right]$, the efficient equilibrium outcome is equally informative regardless of whether evidence is verifiable.*
- *If $b \in \left(b^\circ, \mathbb{E}(\theta)\right)$, the efficient equilibrium outcome is more informative when evidence is verifiable.*
- *If $b \geq \mathbb{E}(\theta)$, the efficient equilibrium outcome is uninformative regardless of whether evidence is verifiable.*

To navigate the result, let us begin by stating a useful intermediate result in the proof of Proposition 5: when F is uniform and (S, π) is uninformative, for every $b \in [0, b^\circ)$, the efficient equilibrium outcome in the disclosure game replicates the outcome of the two-message equilibrium of the cheap-talk game. The disclosure constraint prevents the sender from exploiting more than two on-path actions in that region, despite the availability of $N + 1$ messages in principle. This leads to a couple of immediate implications: First, when $b < \frac{\mathbb{E}(\theta)}{6}$, cheap talk can do strictly better: the sender can sustain equilibria with three or more on-path actions (the exact number depends on b), because incentive constraints are weaker when preferences are closely aligned. Because disclosure is effectively restricted to the two-action benchmark for $b < b^\circ$, the efficient cheap-talk outcome is strictly more informative in this region, proving the first bullet. When instead $b \in \left(\frac{\mathbb{E}(\theta)}{6}, b^\circ\right]$, the efficient outcome in cheap talk collapses to the two-message partition, and the two regimes therefore deliver the same informativeness, proving the second bullet.

The most interesting bullet is perhaps the third one. When $b \in \left(b^\circ, \mathbb{E}(\theta)\right)$, the efficient disclosure outcome becomes strictly more informative than the efficient cheap-talk outcome, even though evidence is completely uninformative. Understanding why this is possible is key to understanding how verifiability constraints can generate informative equilibria in the disclosure game.

To build intuition, let $b = \frac{\mathbb{E}(\theta)}{2}$. For this sender's bias, all equilibria of the cheap talk game are uninformative. By contrast, the disclosure game admits informative equilibria. These equilibria crucially exploit the fact that evidence is noisy and the sender faces verifiability constraints. To see this, consider for instance a generalized cutoff strategy in which the sender discloses s_N if and only if $\theta \geq t_N^*$, and otherwise sends $\circ$. All other disclosures s_i for $i \neq N$ are off path, and we set $\alpha(s_i) = \alpha(\circ)$ for those messages. The proof identifies an interior cutoff $t_N^* \in (0, 1)$ for which this strategy profile can be supported as an equilibrium. Under this equilibrium, actions $\alpha(s_N)$ and $\alpha(\circ)$ are distinct and taken with strictly positive probability. Hence, disclosure sustains an equilibrium whose informativeness remains strictly positive even when the efficient cheap-talk equilibrium is not.

What makes this work is that disclosure embeds an exogenous, verifiable source of randomization: message s_N is available only to senders who actually observe s_N. Even though s_N is uninformative about θ *ex ante*, its stochastic availability limits the sender's ability to mimic across types. This constraint disciplines deviations and allows the sender to sustain informative separation that is not credible in the cheap talk benchmark.

This mechanism is closely related to the idea, familiar from discussions of noisy or mediated communication, that exogenous noise can restore credibility by restricting imitation (see e.g., Myerson (1991), Krishna and Morgan (2004), Blume et al. (2007), Goltsman et al. (2009)).

Here, the noise comes from the realization of verifiable evidence, which acts as a stochastic constraint on feasible messages. In our setting, verifiability matters not because signals directly encode information about θ, but because the verifiability constraints interact with the sender's private knowledge of θ to create credible, informative disclosure.

Beyond the Uniform Distribution. The clean characterization in Proposition 5 relies on the analytical convenience of the uniform distribution. However, as previewed, the main qualitative insight of the discussion above extends to an arbitrary prior distribution F. In particular, we can still show that when b is small, the efficient equilibrium outcome is more informative when evidence is unverifiable; that if $\mathbb{E}(\theta) > \frac{1}{2}$, there is an intermediate range of biases in which the efficient equilibrium outcome is more informative when evidence is verifiable; and, finally, that when the bias is sufficiently large, the efficient equilibrium outcome is uninformative regardless of whether evidence is verifiable. These results are collected in Proposition B.3 in Appendix B.

5 More-Informative Evidence

The previous section showed that the intrinsic informativeness of signals is not essential for the main qualitative results of the paper. It is nevertheless natural to ask what role signal informativeness *does* play. Even if informativeness is not necessary for information transmission, does making evidence more informative (in the Blackwell sense) systematically facilitate information transmission? This section addresses that question.

We begin by introducing an informativeness order on evidence structures. Let (S, π) and (S', π') be two evidence structures, where S and S' are finite signal spaces. We say that (S, π) is Blackwell more informative than (S', π'), written $\pi \succeq \pi'$, if there exists a garbling $\kappa : S \to \Delta(S')$ such that for every $\theta \in \Theta$ and every $s' \in S'$, $\pi'(s' \mid \theta) = \sum_{s \in S} \kappa(s' \mid s)\pi(s \mid \theta)$.

We start with a positive result at the two extreme levels of the sender's bias.

Proposition 6. *Consider two evidence structures (S, π) and (S, π') with $\pi \succeq \pi'$. Suppose $b = 0$ or $b \geq 1$. Then the efficient equilibrium under (S, π) is more informative than the efficient equilibrium under (S, π').*

The argument is immediate when $b \geq 1$. In that case, Proposition 3 shows that the efficient equilibrium outcome is FED, and the informativeness of the FED outcome is monotone in the

Blackwell order: if $\pi \succeq \pi'$, then the posterior induced by π is a mean-preserving refinement of the posterior induced by π', so the corresponding FED outcome is (weakly) more informative.

It is important that this route is unavailable once $b < 1$. Proposition 3 shows that for $b < 1$ the efficient equilibrium need not coincide with FED. Thus, outside the high-bias region, Blackwell monotonicity of the efficient equilibrium cannot be inferred from Blackwell monotonicity of the full-disclosure outcome.

When $b = 0$, the positive result comes from a different simplification: sender and receiver have common interests. As in the proof of Proposition 3, when $b = 0$, the efficient equilibrium can then be characterized through an ancillary commitment problem in which the sender commits ex ante to a verifiable disclosure policy. At first sight one might hope to apply a standard Blackwell simulation argument to this problem. But verifiability prevents that argument from going through directly. After observing a fine signal s, the sender can randomize only over $\{s, \circ\}$; she cannot in general privately simulate a coarse signal s' and then have the receiver condition on that hidden simulated label, because the receiver observes only the actual disclosed fine message. Lemma B.13 resolves this difficulty by first proving monotonicity for deterministic garblings and then extending the result to arbitrary stochastic garblings using two facts: the ancillary value is convex in the experiment, and every stochastic garbling is a mixture of deterministic garblings. It follows that the efficient-equilibrium informativeness is weakly higher under the Blackwell-more-informative evidence structure.

The positive Blackwell-monotonicity results at $b = 0$ and $b \geq 1$ therefore rest on two different simplifications. At $b = 0$, strategic conflict disappears, so more informative evidence translates directly into more informative equilibrium communication via the ancillary commitment problem. At $b \geq 1$, the strategic problem collapses in the opposite direction: sender preferences over receiver actions are effectively type-independent, the efficient equilibrium coincides with FED, and Blackwell monotonicity is immediate. Intermediate values of b are precisely the region in which neither simplification is available. There, equilibrium communication depends jointly on on the informativeness of the evidence and on the state-dependent availability of messages, so a Blackwell improvement can help through the former channel while hurting through the latter.

The next example shows that Blackwell monotonicity can fail for intermediate values of b. To highlight the mechanism as sharply as possible, the example is deliberately extreme and builds on the uninformative-evidence benchmark of Section 4.1.

Let the prior F be uniform and bias be $b = \frac{1}{4}$. We compare two evidence structures: $(S, \pi_{\delta,\varepsilon})$ and $(S, \hat{\pi})$, with $S = \{s_1, s_2\}$. For the former, let

$$\pi_{\delta,\varepsilon}(s_2 \mid \theta) = \begin{cases} \varepsilon & \text{if } \theta \in [0, 1-\delta), \\ 1-\varepsilon & \text{otherwise,} \end{cases} \tag{1}$$

where $\delta, \varepsilon \in (0, \frac{1}{2})$. For the latter, let $\hat{\pi}(s_2 \mid \theta) = 1 - p$, where $p \in (0,1)$. Clearly, $(S, \pi_{\delta,\varepsilon})$ is informative while $(S, \hat{\pi})$ is not, as it is constant in θ. In fact, it is easy to see that the former is Blackwell-more informative than the latter.[7] We will argue that the efficient equilibrium in the disclosure game under $(S, \pi_{\delta,\varepsilon})$ is *less* informative than the efficient equilibrium under $(S, \hat{\pi})$.

Denote by $\overline{V}^{CT2}(b)$ the maximal informativeness over all cheap-talk equilibrium outcomes with at most two on-path messages under the uniform prior. Since $b = \frac{1}{4}$, every such cheap-talk equilibrium is babbling, and thus $\overline{V}^{CT2}(b) = 0$ (see Figure 2, or Lemma B.5). Next, observe that for any $\eta > 0$ there exist δ and ε sufficiently small such that the informativeness of the efficient equilibrium under $(S, \pi_{\delta,\varepsilon})$ is at most $\overline{V}^{CT2}(b) + \eta = \eta$.[8] In other words, the informativeness of the efficient equilibrium under $(S, \pi_{\delta,\varepsilon})$ can be made arbitrarily small.

By contrast, Proposition 5 shows that, at $b = \frac{1}{4}$, the efficient equilibrium of the disclosure game under $(S, \hat{\pi})$ is strictly informative. Therefore, for sufficiently small δ and ε, the efficient equilibrium of the disclosure game under $(S, \pi_{\delta,\varepsilon})$ is less informative than the efficient equilibrium under $(S, \hat{\pi})$, even though $\pi_{\delta,\varepsilon} \succeq \hat{\pi}$. Therefore, we obtain an example in which a Blackwell-more-informative evidence structure leads to an efficient equilibrium outcome that is less informative than the one induced by a Blackwell-less-informative evidence structure. This failure is not special to $b = \frac{1}{4}$, but holds for all $b \in (b^\circ, \frac{1}{2})$, where b° is as in Proposition 5.

What drives this reversal? In line with the message of Section 4.1, this example clarifies how the informativeness of the efficient equilibrium is driven not only by how informative evidence

[7] To see this, let κ be the garbling defined by $\kappa(s_1 \mid s_1) = \kappa(s_1 \mid s_2) = p$ and $\kappa(s_2 \mid s) = 1 - p$ for all $s \in S$. Then for every θ, $\sum_s \kappa(s_1 \mid s)\pi_{\delta,\varepsilon}(s \mid \theta) = p = \hat{\pi}(s_1|\theta)$.

[8] A more general result delivering this conclusion is proved in Lemma B.14. Here we simply provide a partial intuition. If (θ, s) is drawn from the joint distribution induced by $(F, S, \pi_{\delta,\varepsilon})$, the feasible message set equals $\{s_1, \circ\}$ with probability $\Pr(s_1) = (1-\delta)(1-\varepsilon) + \delta\varepsilon$, which converges to 1 as $(\delta, \varepsilon) \to (0, 0)$. Thus rare s_2-types become asymptotically negligible, and the disclosure game under $(S, \pi_{\delta,\varepsilon})$ almost surely coincides with a two-message cheap-talk game.

is, but also by how *flexibly* the sender can use the available language. In a disclosure game, flexibility is governed by the feasibility constraints induced by π: message s_i is available only to types who actually observe s_i. Thus, what matters for equilibrium information transmission is not only how posteriors vary with s, but also how often different messages are *available* across types.

In the example with $(S, \pi_{\delta,\varepsilon})$, message s_2 is highly informative about high states, but it is also almost never available: $\Pr(s_2) = \varepsilon + \delta - 2\delta\varepsilon \to 0$. As $(\delta, \varepsilon) \to (0, 0)$, with probability approaching 1 the sender's feasible message set is effectively $\{\circ, s_1\}$, so the disclosure game approaches a two-message cheap-talk environment. At $b = \frac{1}{4}$, that limiting benchmark is babbling. Under $(S, \hat{\pi})$, by contrast, both evidence messages remain available with strictly positive probability even though the signal itself is statistically uninformative. This balanced stochastic availability of verifiable messages is valuable: it constrains imitation while preserving a nondegenerate verifiable language, and can therefore sustain informative disclosure.

The broader implication is that, for intermediate bias levels, Blackwell informativeness is not a sufficient statistic for the value of evidence in a disclosure game. What matters is the combination of the statistical content of evidence and the feasibility constraints it induces. More informative evidence can improve the former while worsening the latter, and the net effect on equilibrium informativeness can therefore be negative. Seen this way, the negative result in this section is not about "information being bad." It is about the fact that, in disclosure games, a change in the evidence structure also changes the sender's effective verifiable language.

6 Final Remarks

A richer set of verifiable statements. Throughout the paper, our model assumes that the message set available to type (θ, s) is $M^V(\theta, s) = \{\circ, s\}$: the sender either discloses s or remains silent. A natural generalization, which nests our model, lets $M^V(\theta, s) = \{m \in 2^S : s \in m\}$, so the sender may disclose any subset of S that contains the realized signal s. This allows a richer set of verifiable claims: not only "I observed s," but also statements such as "I did not observe s" or "my signal was higher than s." This added richness does not change the paper's main qualitative conclusions. In particular, for small b, FED equilibria still fail to exist, and there remain equilibria that are more informative than the FED outcome. A richer verifiable language simply gives the sender additional *flexibility* and can sustain higher informativeness when preferences are sufficiently aligned. At the same time, as long as S is finite, the verifiable language is still finite; hence, for sufficiently small b, the efficient disclosure outcome remains less informative than its cheap-talk counterpart. By contrast, when b is large, the added flexibility does not undermine credibility: the FED outcome remains attainable. Put differently, enlarging the set of verifiable statements attenuates the flexibility–credibility trade-off without overturning it. Our baseline binary-disclosure model has the virtue of isolating this tradeoff in its simplest and starkest form.

Interacting Verifiable and Unverifiable Communication. Our disclosure model has another feature worth highlighting: it allows verifiable and unverifiable communication to coexist within the same environment. Arguably, this mode of communication is common in practice. For instance, financial disclosure (e.g., earnings reports) typically involve the disclosure of hard facts (sales in the previous quarter) with unverifiable statements (predictions about the future). There is relatively little work that studies the interaction of disclosure and cheap talk.[9] In part, this lack of attention is due to the fact that the classic disclosure model leaves little room for cheap talk. When the sender can verifiably reveal all of her private information, a FED equilibrium exists for all b and achieves the first best (see Proposition B.1). In that case, disclosure exhausts communication and there is no scope for cheap talk.

Our model, by contrast, creates scope for such interaction because disclosure does not exhaust what the sender knows. The interplay between cheap talk and verifiable disclosure is promising direction for future research in this literature and our disclosure framework provides a natural starting point for that agenda.

[9] Bertomeu and Marinovic (2016) were the first to study this interaction. They consider a model in which the sender has two distinct pieces of information, only one of which is verifiable. In their model, disclosure is costly, the sender's preferences are strictly increasing in the receiver's action (maximal bias), and there is uncertainty about the sender's ability to use cheap talk. More recently, Dasgupta (2023) examines a model where the sender has state-independent preferences, can verifiably reveal the state, and can also send cheap talk messages to an uninformed receiver. She identifies a curvature condition on the sender's payoff that guarantees a full-disclosure equilibrium, where cheap talk is irrelevant. When this condition fails, she shows that cheap talk may prevent unraveling.

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Appendix

A Extra Material

A.1 Equilibrium Definition

Definition 4 (PBE)**.** *We define four properties of an assessment* (σ, α, μ)*:*

1. *(Receiver optimality) For every message* m*,* $\alpha(m) = \mathbb{E}_{\mu}[\theta|m]$*;*

2a. *(Sender optimality with verifiable evidence) For every sender's type* (θ, s) *and for any message* m*,*

$$\sigma(m|\theta, s) > 0 \quad \Longrightarrow \quad m \in \arg\max_{m' \in \{\circ, s\}} u_S(\alpha(m'), \theta)$$

2b. *(Sender optimality with unverifiable evidence) For every sender's type* (θ, s) *and for any message* m*,*

$$\sigma(m|\theta, s) > 0 \quad \Longrightarrow \quad m \in \arg\max_{m' \in M} u_S(\alpha(m'), \theta)$$

3 *(Belief Consistency). Beliefs are pinned down from Bayes' rule whenever possible.*

An assessment is a Perfect Bayesian Equilibrium of the game with verifiable evidence if it satisfies property 1, 2a and 3. It is an equilibrium of the game with unverifiable evidence if it satisfies properties 1, 2b, and 3.

A.2 Cheap Talk Equivalence

As discussed in the main text, our game with unverifiable evidence is a rather standard cheap-talk game with a restriction on the message alphabet: the sender can send at most $N + 1$ messages in equilibrium (the cardinality of M). Note that the signal structure (S, π) plays no role beyond determining the size of the alphabet.

To see this, note that since the sender can choose any message $m \in M \triangleq S \cup \{\circ\}$ regardless of the realized evidence s, and since payoffs depend on (a, θ) but not on s, the fact that the signal realizations are actually informative is strategically inessential. Formally, for any sender strategy $\sigma : \Theta \times S \to \Delta(M)$ in the game with unverifiable evidence define its reduced form $\hat{\sigma} : \Theta \to \Delta(M)$ by $\hat{\sigma}(m \mid \theta) = \sum_{s \in S} \pi(s \mid \theta)\sigma(m \mid \theta, s)$. Note that $\hat{\sigma}$ is a standard cheap-talk strategy. For any receiver strategy $\alpha : M \to A$, the induced distribution of (θ, m) and the receiver posterior $\mu(\theta \mid m)$ depend on σ only through $\hat{\sigma}$. Moreover, every PBE outcome of the unverifiable-evidence game can be replicated by a PBE in which the sender ignores evidence and plays $\sigma(m \mid \theta, s) \triangleq \hat{\sigma}(m \mid \theta)$. The other direction trivially holds. Consequently, the unverifiable-evidence game is outcome-equivalent to a standard cheap-talk game with message space M.

B Proofs

B.1 Proofs for Section 3

Lemma B.1. *Let $b < \frac{\mathbb{E}(\theta|s_N)+\mathbb{E}(\theta|s \neq s_N)}{2}$. Then the disclosure game admits an informative equilibrium.*

Proof of Lemma B.1. Let $p_N(\theta) \triangleq \Pr(s = s_N \mid \theta) = \pi(s_N \mid \theta)$. We construct a class of sender strategies indexed by $t \in [0, 1]$: the sender discloses the evidence s if and only if $s = s_N$ and $\theta \geq t$; otherwise the message $m = \circ$ is sent. Conditional on observing $m = s_N$, the receiver infers $(s = s_N, \theta \geq t)$, hence $a_N(t) \triangleq \mathbb{E}[\theta \mid m = s_N] = \mathbb{E}[\theta \mid s = s_N, \theta \geq t]$. By continuity, define $a_N(1) \triangleq 1$. Conditional on θ, the likelihood of observing $m = \circ$ is

$$\Pr(m = \circ \mid \theta) = \begin{cases} 1 & \theta < t \\ 1 - p_N(\theta) & \theta \geq t. \end{cases}$$

Thus the posterior mean is

$$a_0(t) \triangleq \mathbb{E}[\theta \mid m = \circ] = \frac{\int_0^t \theta f(\theta)d\theta + \int_t^1 \theta(1 - p_N(\theta))f(\theta)d\theta}{F(t) + \int_t^1 (1 - p_N(\theta))f(\theta)d\theta}.$$

which can be equivalently written as a convex combination

$$a_0(t) = \gamma \cdot \mathbb{E}[\theta \mid \theta \leq t] + (1 - \gamma) \cdot \mathbb{E}[\theta \mid \theta \geq t, s \neq s_N].$$

where $\gamma \triangleq \frac{F(t)}{F(t)+\int_t^1 (1-p_N(\theta))f(\theta)d\theta}$. By MLRP, truncation preserves the ranking of posterior means, hence $\mathbb{E}[\theta \mid s = s_N, \theta \geq t] \geq \mathbb{E}[\theta \mid s \neq s_N, \theta \geq t]$. Also note that $\mathbb{E}[\theta \mid \theta \leq$

$t] < \mathbb{E}[\theta \mid s = s_N, \theta \geq t]$. Therefore, for all $t \in [0,1]$, $a_N(t) \geq a_0(t)$, and in particular if $t \in (0,1)$, $\gamma \in (0,1)$ and, thus, $a_N(t) > a_0(t)$. The disclosure behavior induced by threshold t is incentive compatible for the sender of type (θ, s_N) if

$$\theta \geq \frac{a_N(t) + a_0(t)}{2} - b.$$

We want to find a t such that, type $(\theta = t, s_N)$ satisfies is indifferent between disclosing or not, namely it satisfies the equation above with equality. Let

$$h(t) \triangleq \frac{a_N(t) + a_0(t)}{2} - b - t.$$

Note that $a_N(t)$ and $a_0(t)$ are continuous in t, hence h is continuous on $[0,1]$. At $t = 0$, $a_N(0) = \mathbb{E}[\theta \mid s = s_N]$ and $a_0(0) = \mathbb{E}[\theta \mid s \neq s_N]$, and therefore

$$h(0) = \frac{\mathbb{E}[\theta \mid s = s_N] + \mathbb{E}[\theta \mid s \neq s_N]}{2} - b > 0,$$

by the assumption of the lemma. At $t = 1$, instead, disclosure occurs with probability 0, so $a_0(1) = \mathbb{E}[\theta] \triangleq \mu$, and by definition $a_N(1) = 1$. Thus

$$h(1) = \frac{1+\mu}{2} - b - 1 = \frac{\mu - 1}{2} - b < 0$$

because $\mu < 1$ (full support) and $b \geq 0$. By the intermediate value theorem, there exists an interior $t^* \in (0,1)$ such that $h(t^*) = 0$, that is t^* is an equilibrium cutoff.

Finally, specify an assessment as follows: the sender plays the above strategy with index t^*; the receiver plays $a_N(t^*)$ after $m = s_N$ and $a_0(t^*)$ after $m = \circ$. After any off-path message, let the receiver hold any belief with mean $a_0(t^*)$ and choose action $a_0(t^*)$. Receiver optimality holds because actions equal posterior means. Sender optimality holds because $h(t^*) = t^*$. Moreover, since $t^* \in (0,1)$ and $a_N(t^*) > a_0(t^*)$, both $m = s_N$ and $m = \circ$ occur with positive probability and induce different actions, so the equilibrium is strictly informative.

In particular, if π is uninformative then $\mathbb{E}[\theta \mid s = s_N] = \mathbb{E}[\theta \mid s \neq s_N] = \mathbb{E}[\theta]$, so the condition in the statement reduces to $b < \mathbb{E}[\theta]$. □

Proof of Proposition 1. Fix $b \geq 0$. We begin by constructing an uninformative equilibrium. Let $\bar{\theta} \triangleq \mathbb{E}[\theta]$. Consider the following assessment (σ, α, μ): $\sigma(m|\theta, s) = \mathbb{1}\{m = \circ\}$, for all $(\theta, s) \in \Theta \times S$; $\alpha(m) = \bar{\theta}$, for all $m \in M$; $\mu(\cdot|m)$ is given by Bayes' rule if $m = \circ$. Since $m = \circ$ is sent with probability one for every type, $\alpha(\circ) = [\theta|m = \circ] = \bar{\theta}$. For any off-path message $m = s$, define $\mu(\cdot|s)$ arbitrarily subject only to having mean $\bar{\theta}$. This ensures receiver's optimality and the belief consistency. Finally, sender's optimality is trivial since all messages lead to the same action. Therefore, the assessment constitutes a PBE.

Next suppose evidence is informative. By Lemma B.1, we know an informative equilibrium exists if $b < \frac{1}{2}\mathbb{E}(\theta \mid s_N) \leq \frac{1}{2}(\mathbb{E}(\theta \mid s_N) + \mathbb{E}(\theta \mid s \neq s_N))$. By Proposition 2, we know a FED equilibrium exists if $b \geq \frac{1}{2}\mathbb{E}(\theta \mid s_N)$. Since evidence is informative, the FED equilibrium is informative. Therefore, there always exists an informative equilibrium. □

Proof of Lemma 1. Fix a PBE $(\sigma^*, \alpha^*, \mu^*)$. Write $a_0 \triangleq \alpha^*(\circ)$ and $a_i \triangleq \alpha^*(s_i)$ for all i. If $\Pr_{\sigma^*}(m = \circ) = 0$, then the sender never uses silence and thus discloses every signal for every θ. Thus, the sender's strategy is already of the desired form: take $H = \{1, \ldots, N\}$, $L = M = \emptyset$, and $t_i = 0$ for all i. Hence assume $\Pr_{\sigma^*}(m = \circ) > 0$. Then $a_0 \in (0, 1)$. Let

$$I_0 \triangleq \left\{ i \in \{1, \ldots, N\} : \Pr_{\sigma^*}(m = s_i) > 0 \text{ and } a_i = a_0 \right\}.$$

Construct an assessment $(\hat{\sigma}, \hat{\alpha}, \hat{\mu})$ as follows: $\hat{\alpha} = \alpha^*$; for every $i \in I_0$, set $\hat{\sigma}(\circ \mid \theta, s_i) = 1$ for all θ; and for every $i \notin I_0$, set $\hat{\sigma}(\cdot \mid \theta, s_i) = \sigma^*(\cdot \mid \theta, s_i)$ for all θ. Let $\hat{\mu}$ be given by Bayes' rule after every message m with $\Pr_{\hat{\sigma}}(m) > 0$, and choose $\hat{\mu}(\cdot \mid m)$ arbitrarily after any off-path message subject to $E_{\hat{\mu}}[\theta \mid m] = \hat{\alpha}(m)$. It is easy to see that $(\hat{\sigma}, \hat{\alpha}, \hat{\mu})$ is a PBE and is outcome-equivalent to $(\sigma^*, \alpha^*, \mu^*)$. Indeed, the only change is that whenever $i \in I_0$ the sender now sends $\circ$ instead of sometimes sending s_i, but by assumption both messages induce the same receiver action a_0, hence the realized action (and therefore the joint distribution of (θ, a)) is unchanged. Receiver optimality after $\circ$ is also preserved: under $(\sigma^*, \alpha^*, \mu^*)$ we have $\mathbb{E}[\theta \mid m = \circ] = a_0$ and for each $i \in I_0$ $\mathbb{E}[\theta \mid m = s_i] = a_0$; therefore pooling those s_i-events into $\circ$ does not change the receiver's action. Thus, it is without loss of generality to assume $I_0 = \emptyset$. From now on, work with $(\hat{\sigma}, \hat{\alpha}, \hat{\mu})$ and keep the notation a_0 and $a_i = \hat{\alpha}(s_i)$.

Fix an index i such that $\Pr_{\hat{\sigma}}(m = s_i) > 0$. By construction, $a_i \neq a_0$. When the sender's evidence is s_i, disclosing s_i is optimal iff $\theta \leq t_i$ when $a_i < a_0$ and $\theta \geq t_i$ when $a_i > a_0$, where

$$t_i \triangleq \max\left\{0, \min\left\{1, \frac{a_i + a_0}{2} - b\right\}\right\} \in [0, 1].$$

Note that, since F is atomless, we can take the sender's behavior after each such s_i to be deterministic one-sided cutoff behavior. Define

$$L \triangleq \left\{ i : \Pr_{\hat{\sigma}}(m = s_i) > 0 \text{ and } a_i < a_0 \right\}, \quad H \triangleq \left\{ i : \Pr_{\hat{\sigma}}(m = s_i) > 0 \text{ and } a_i > a_0 \right\},$$

and let $M \triangleq \{1, \dots, N\} \setminus (L \cup H)$. Let σ' be a generalized cutoff strategy given (L, M, H) and let $(t_i)_{i \in L \cup H} \subseteq [0,1]$ be the projected cutoff defined above. Define $\alpha' \triangleq \hat{\alpha}$ and $\mu' \triangleq \hat{\mu}$. It is immediate to see that (σ', α', μ') is a PBE that is outcome-equivalent to the original one, and in which the sender uses a generalized cutoff strategy. □

Lemma B.2. *Fix a PBE (σ, α, μ) that induces the FED outcome. Without loss of generality, we have that either $m = \circ$ is off path, or else $m = \circ$ is sent by type (θ, s_i) if and only if $i \in M$, and moreover $\alpha(\circ) = a_i$ for all $i \in M$.*

Proof. Fix a PBE (σ, α, μ) that induces the FED outcome. By Lemma 1, let (σ', α', μ') be an outcome-equivalent PBE in which σ' is a generalized cutoff rule, with partition (L, M, H) and cutoffs $(t_i)_{i \in L \cup H}$. Write $a_\circ \triangleq \alpha'(\circ)$ and $a_i \triangleq \mathbb{E}[\theta \mid s_i]$ for all i. We show that we can pick $L = \emptyset$ and $t_i = 0$ for all $i \in H$. Moreover, $a_i = a_\circ$ for all $i \in M$. This implies that, if $a_i \neq a_j$, they cannot both belong to M.

To see this, let $i \in H$. If $t_i \in (0,1)$, then both events $\{\theta \geq t_i\}$ and $\{\theta < t_i\}$ have strictly positive probability conditional on s_i. Therefore,

$$\mathbb{E}[\theta \mid s_i] = \Pr(\theta \geq t_i \mid s_i) \cdot \mathbb{E}[\theta \mid s_i, \theta \geq t_i] + \Pr(\theta < t_i \mid s_i) \cdot \mathbb{E}[\theta \mid s_i, \theta < t_i].$$

Since $\mathbb{E}[\theta \mid s_i, \theta \geq t_i] > \mathbb{E}[\theta \mid s_i, \theta < t_i]$ and both weights are strictly positive, we obtain $\mathbb{E}[\theta \mid s_i, \theta \geq t_i] > \mathbb{E}[\theta \mid s_i] = a_i$. But since $m = s_i$ is on path (since $i \notin M$) then Bayes' rule pins down $\mu'(\cdot \mid s_i)$ and receiver optimality implies $\alpha'(s_i) = \mathbb{E}[\theta \mid m = s_i] = \mathbb{E}[\theta \mid s_i, \theta \geq t_i] > a_i$. Hence $\alpha'(s_i) > a_i$, contradicting that the outcome is FED. Thus $t_i \notin (0,1)$ and, since $i \in H$, we must have $t_i = 0$. A similar argument can be used to show that if $i \in L$, $t_i = 1$. But then it is without loss of generality to take $L = \emptyset$. Consequently, if $m = \circ$ is on path, it can only be sent by types (θ, s_i) such that $i \in M$. Therefore, $M = \emptyset$ if and only if $m = \circ$ is off path. Finally, if $\circ$ is on path then $a_\circ = \mathbb{E}(\theta \mid s_i)$ for every $i \in M$, since the outcome is FED. Since the evidence is nontrivial, this is possible only if $|M| \leq 1$. □

Proof of Proposition 2. *(Necessity).* Suppose there exists a PBE (σ, α, μ) that induces the FED outcome. Let $a_\circ \triangleq \alpha(\circ)$ and $a_i \triangleq \mathbb{E}(\theta \mid s_i)$. Since evidence is informative, we have $a_1 < a_N$. By Lemma 1, there is no loss in restricting attention to generalized cutoff sender strategies and by Lemma B.2 the sender's strategy σ is such that either $m = \circ$ is off path, or else $m = \circ$ is sent by type (θ, s_i) if and only if $i \in M$, and moreover $\alpha(\circ) = a_i$ for all $i \in M$.

Consider type $(\theta, s) = (1, s_1)$, which occurs with positive probability. In a FED equilibrium, this type induces action a_1. If she deviates to $\circ$, the induced action is $a_\circ$. Since her payoff $v(a, 1)$ is strictly increasing in a over all θ, she prefers higher actions. Therefore, sequential rationality at $(1, s_1)$ implies $a_\circ \leq a_1 c$ and thus $a_\circ \leq a_1 < a_N$.

Consider now type $(\theta, s) = (0, s_N)$. In a FED equilibrium, this type induces action a_N. If she deviates to $\circ$, the induced action is $a_\circ$. Sequential rationality at $(0, s_N)$ implies $(a_N - b)^2 \leq (a_\circ - b)^2$. Since $a_N > a_\circ$, this inequality is equivalent to

$$b \geq \frac{a_N + a_\circ}{2} \geq \frac{a_N}{2} = \frac{1}{2}\mathbb{E}[\theta \mid s_N] \triangleq \bar{b}.$$

This proves necessity.

Sufficiency. Assume $b \geq \bar{b}$. Construct an assessment (σ, α, μ) as follows. The sender always discloses the evidence: $\sigma(m \mid \theta, s) = \mathbb{1}\{m = s\}$. Let $\alpha(s_i) = a_i = \mathbb{E}[\theta \mid s_i]$ for all i and $\alpha(\circ) = 0$. Beliefs after each s_i are given by Bayes' rule; beliefs after $\circ$ are chosen so that $\mathbb{E}[\theta \mid \circ] = 0$. Receiver sequential rationality holds by quadratic loss. For sender optimality, fix any type (θ, s_i) and let $x \triangleq \theta + b$. Since $\theta \geq 0$,

$$x \geq b \geq \bar{b} = \frac{a_N}{2} \geq \frac{a_i}{2},$$

which implies $(a_i - x)^2 \leq x^2$. Thus, disclosing s_i (inducing a_i) weakly dominates deviating to $\circ$ (inducing $a_0 = 0$). Hence the assessment is a PBE and in particular a FED. □

Proof of Proposition 3.

First Clause: *Efficiency of FED for* $b \geq 1$. By Proposition 2, we know a FED equilibrium exists when $b \geq 1$. We are left to show that the FED outcome is the most informative outcome that can be supported in a PBE. Fix $b \geq 1$ and let (σ, α, μ) be any PBE. For each i let $a_i = \alpha(s_i)$ and $a_0 = \alpha(\circ)$. Fix $s = s_i$. Verifiability implies the sender's feasible messages are $\{s_i, \circ\}$, inducing actions $\{a_i, a_0\}$. Since $b \geq 1$, the sender always prefers higher receiver actions, independently of θ. Therefore, if $a_i > a_0$, $\sigma(s_i|\theta, s_i) = 1$ for all θ; If $a_i < a_0$, $\sigma(s_i|\theta, s_i) = 0$ for all θ; If $a_i = a_0$, then the sender is indifferent between s_i and $\circ$, so $\sigma(s_i|\theta, s_i)$ can be arbitrary in θ.

Let M^* denote the set of on-path messages under σ. Partition M^* into level sets of the receiver's equilibrium action $\alpha(m)$. Fix any level set $C \subseteq M^*$ such that $\alpha(m) = \bar{a}$ for all $m \in C$. We have $\bar{a} = \alpha(m) = \mathbb{E}[\theta|m]$ for all $m \in C$. Let $\hat{m}$ be the coarsening of m that replaces all messages in C by a single label $\hat{m} = C$ and leaves all other messages unchanged. Then

$$\mathbb{E}[\theta|\hat{m} = C] = \mathbb{E}[\mathbb{E}[\theta|m]|\hat{m} = C] = \bar{a},$$

so the receiver's best reply after $\hat{m} = C$ is still $\bar{a}$. Moreover, this change leaves the receiver's ex ante payoff unchanged. Thus, for the purpose of maximizing the receiver's payoff over equilibria, we may without loss assume (σ, α, μ) is such that on-path messages induce distinct actions.

Since the only possible messages are $\circ$ or s_i, there are only two kinds of on-path messages: Message s_i with $a_i > a_0$; and one pooled message (call it $\circ$) collecting all remaining on-path messages, each of which induces action a_0. Let $J = \{i \in \{1, \ldots, N\} : a_i > a_0\}$. For each $i \in J$, we have $\sigma(s_i|\theta, s_i) = 1$ for all θ, so the event $\{m = s_i\}$ coincides with the event $\{s = s_i\}$. Hence $a_i = \alpha(s_i) = \mathbb{E}[\theta|s = s_i]$ for all $i \in J$. In other words, the merged message function is a garbling of s. Therefore, in every PBE with $b \geq 1$ the receiver's ex ante payoff is weakly smaller than under mandated disclosure.

Second Clause: *Inefficiency of FED for $b < 1$ and special (F, S, π).* Suppose $b < 1$. We construct an environment (F, S, π) in which there exists an equilibrium that is more informative than the FED outcome. Suppose the evidence structure (S, π) is uninformative. Then the FED outcome is uninformative. By Lemma B.1, an informative equilibrium—and hence an equilibrium that is more informative than the FED outcome—exists if $b < \frac{\mathbb{E}(\theta|s_N) + \mathbb{E}(\theta|s \neq s_N)}{2} = \mathbb{E}(\theta) \in (0, 1)$. Let F be any prior such that $\mathbb{E}(\theta) > b$ and note that such an F can be found since $b < 1$.

Third Clause: *Inefficiency of FED for any (F, S, π) and small b* . Follows from Lemmas B.3 and B.4. □

Lemma B.3. *Fix $b = 0$. The efficient equilibrium outcome is strictly more informative than mandated disclosure.*

Proof of Lemma B.3. We proceed in the following steps:

Step 1: An ancillary problem. Fix $b = 0$. Let λ denote the joint distribution of (θ, s) induced by (F, π). Let P be the set of all probability measures φ on $[0, 1] \times S \times M$ such that (i) the marginal of φ on (θ, s) equals λ, and (ii) verifiability holds: $\varphi\big(\{(\theta, s, m) : m \notin \{s, \circ\}\}\big) = 0$.

Let $A \triangleq [0,1]^M$ be the set of all receiver action rules $a : M \to [0,1]$. For $(\varphi, a) \in P \times A$ define

$$W(\varphi, a) \triangleq -\int (a(m) - \theta)^2 \, d\varphi(\theta, s, m).$$

(When $b = 0$, maximizing the receiver payoff is equivalent to maximizing social welfare, up to a constant factor.)

Because $[0,1] \times S \times M$ is compact, the set of Borel probability measures on it is compact in the weak topology. The constraints defining P are closed, hence P is compact. Also A is compact and W is continuous. Therefore, by Weierstrass, there exists a maximizer $(\varphi^\star, a^\star) \in P \times A$.

Step 2: The maximizer induces a PBE. We show that the maximizer $(\varphi^\star, a^\star)$ induces a PBE.

Receiver optimality. Fix $\varphi^\star$. Conditional on any message m with $\varphi^\star(m) > 0$, the term

$$-\int (a(m) - \theta)^2 \, d\varphi^\star(\theta, s \mid m)$$

is uniquely maximized at $a(m) = \mathbb{E}_{\varphi^\star}[\theta \mid m]$. Because the objective is separable across messages, at the maximizer,

$$a^\star(m) = \mathbb{E}_{\varphi^\star}[\theta \mid m] \qquad \text{for every } m \text{ with } \varphi^\star(m) > 0.$$

For messages m with $\varphi^\star(m) = 0$, the choice of $a^\star(m)$ does not affect $W(\varphi^\star, a^\star)$.

Disintegrate $\varphi^\star$ into a sender strategy. Since $\varphi^\star$ has marginal λ on (θ, s), there exists a measurable kernel $\sigma^\star(\cdot \mid \theta, s) \in \Delta(M)$ such that

$$\varphi^\star(d\theta, ds, dm) = \lambda(d\theta, ds)\sigma^\star(dm \mid \theta, s).$$

Concretely, $\sigma^\star(\cdot \mid \theta, s)$ can be taken as a regular conditional distribution of m given (θ, s) under $\varphi^\star$. The verifiability constraint on $\varphi^\star$ implies that $\sigma^\star(\cdot \mid \theta, s)$ is supported on $\{s, \circ\}$ for λ-almost every (θ, s); modifying $\sigma^\star$ on a λ-null set if needed, we may assume this support condition holds for every (θ, s).

Beliefs. Define $\mu^\star(\cdot \mid m)$ by Bayes' rule on-path: for every m with $\varphi^\star(m) > 0$ set $\mu^\star(\cdot \mid m) = \varphi^\star(\cdot \mid m)$. For off-path messages m, choose any belief $\mu^\star(\cdot \mid m)$ consistent with feasibility (in particular, if $m = s_i$ then the belief puts probability 1 on $s = s_i$) and such that $\mathbb{E}_{\mu^\star}[\theta \mid m] = a^\star(m)$. This is always possible since $\theta \in [0,1]$ and $a^\star(m) \in [0,1]$.

Then the receiver's strategy $\alpha^\star \triangleq a^\star$ satisfies receiver sequential rationality: $\alpha^\star(m) = \mathbb{E}_{\mu^\star}[\theta|m]$ for all m.

Sender optimality. Fix $\alpha^\star$. For each type (θ, s), define a (measurable) pointwise best reply $\bar{\sigma}(\cdot \mid \theta, s)$ by choosing

$$m(\theta, s) \in \arg\max_{m \in \{s, \circ\}} -(\alpha^\star(m) - \theta)^2,$$

and setting $\bar{\sigma}(m(\theta, s) \mid \theta, s) = 1$ (break ties measurably). Let $\bar{\varphi}$ be the induced law:

$$\bar{\varphi}(d\theta, ds, dm) = \lambda(d\theta, ds)\bar{\sigma}(dm \mid \theta, s).$$

Then $\bar{\varphi} \in P$, and pointwise optimality implies

$$W(\bar{\varphi}, \alpha^\star) \geq W(\varphi^\star, \alpha^\star) = W(\varphi^\star, a^\star).$$

But $(\varphi^\star, a^\star)$ maximizes W over $P \times A$, so equality must hold. If $\sigma^\star$ assigned positive probability to a strictly suboptimal message for a set of types of positive λ-measure, the inequality would be strict, a contradiction. Hence $\sigma^\star$ is (after a null-set modification) a best response to $\alpha^\star$ for every type. Belief consistency holds by construction. Therefore $(\sigma^\star, \alpha^\star, \mu^\star)$ is a PBE.

Step 3: Strict dominance over mandated disclosure. Under quadratic loss and best responses, the receiver's ex ante payoff equals

$$\mathbb{E}\big[u(\alpha(m), \theta)\big] = -\mathbb{E}\big[(\alpha(m) - \theta)^2\big] = -\mathbb{E}\big[\operatorname{Var}(\theta \mid m)\big], \tag{2}$$

because $\alpha(m) = \mathbf{E}[\theta \mid m]$ for every on-path message m.

Let $(\sigma^{MD}, \alpha^{MD})$ be mandated disclosure (Definition 2), and let U^{MD} be the receiver's ex ante payoff under that outcome. Fix any cutoff $\bar{t} \in (0,1)$ and define a feasible (verifiable) sender strategy $\tilde{\sigma}$ by:

$$\tilde{\sigma}(m = s \mid \theta, s) = 1 \quad \text{for all } (\theta, s) \text{ with } s \neq s_N,$$

and for $s = s_N$,

$$\tilde{\sigma}(m = \circ \mid \theta, s_k) = \mathbb{1}\{\theta < \bar{t}\}, \qquad \tilde{\sigma}(m = s_k \mid \theta, s_k) = \mathbb{1}\{\theta \geq \bar{t}\}.$$

Let $\tilde{\varphi}$ be the induced joint law of (θ, s, m) and let $\tilde{\alpha}(m) = \mathbb{E}_{\tilde{\varphi}}[\theta \mid m]$ be the receiver best reply.

Relative to mandated disclosure, the only change is that conditional on $s = s_N$ the information is refined into two messages that split the conditional distribution of θ into $\{\theta < \bar{t}\}$ and $\{\theta \geq \bar{t}\}$. Since F is atomless with full support and $\pi(s_N \mid \theta) > 0$ for all θ, both events have strictly positive probability conditional on s_N, and their conditional means differ:

$$\mathbb{E}[\theta \mid s_N, \theta < \bar{t}] < \mathbb{E}[\theta \mid s_N, \theta \geq \bar{t}].$$

Applying the law of total variance conditional on s_N,

$$\mathrm{Var}(\theta \mid s_N) = \mathbb{E}[\mathrm{Var}(\theta \mid m) \mid s_N] + \mathrm{Var}(\mathbb{E}[\theta \mid m] \mid s_N),$$

and the last term is strictly positive, implying

$$\mathbb{E}[\mathrm{Var}(\theta \mid m) \mid s_N] < \mathrm{Var}(\theta \mid s_N).$$

For $s \neq s_N$, posteriors are unchanged. Hence the ex ante expected posterior variance is strictly lower under $(\tilde{\sigma}, \tilde{\alpha})$ than under mandated disclosure, so the receiver's expected payoff is strictly higher:

$$W(\tilde{\varphi}, \tilde{\alpha}) \ > \ U^{MD}.$$

Since $(\varphi^\star, a^\star)$ maximizes W over $P \times A$ and $(\tilde{\varphi}, \tilde{\alpha}) \in P \times A$, we have

$$W(\varphi^\star, a^\star) \ \geq \ W(\tilde{\varphi}, \tilde{\alpha}) \ > \ U^{MD}.$$

Because $b = 0$, the receiver payoff in the welfare-maximizing PBE constructed in Step 2 equals $W(\varphi^\star, a^\star)$, and therefore it is strictly larger than U_R^{MD}. □

Let $(\sigma^\star, \alpha^\star, \mu^\star)$ be the $b = 0$ equilibrium delivered by Lemma B.3. Since this equilibrium yields the receiver a strictly higher payoff than mandated disclosure, it must satisfy $\Pr_{\sigma^\star}(m = \circ) > 0$, so the silence action $a_0^\star \triangleq \alpha^\star(\circ) = \mathbb{E}[\theta \mid m = \circ]$ is well defined and lies in $(0, 1)$. By Lemma 1, we may represent this equilibrium, without loss of outcome, by a generalized cutoff rule $(L^\star, M^\star, H^\star, t^\star)$ constructed in the proof of that lemma. Under the construction,

$$L^\star \cup H^\star = \{i : \Pr_{\sigma^\star}(m = s_i) > 0\},$$

so $L^\star \cup H^\star$ is exactly the set of on-path evidence messages; moreover, $a_i \neq a_0$ for every $i \in L^\star \cup H^\star$ and hence $t^\star \in (0,1)^{L^\star \cup H^\star}$. Let $J \triangleq L^\star \cup H^\star$.

For each $t \in (0,1)^J$, let $(a_i(t))_{i\in J}$ and $a_0(t)$ denote the receiver's posterior-mean actions induced by the generalized cutoff rule with the same partition $(L^\star, M^\star, H^\star)$ and cutoffs t on the coordinates in J (messages in $M^\star$ are never disclosed). For each $b \geq 0$ and each $t \in (0,1)^J$, define the active-coordinate indifference map $\Phi_b^J : (0,1)^J \to \mathbb{R}^J$ by

$$\Phi_b^J(t) \triangleq \left(\tfrac{1}{2}\big(a_i(t) + a_0(t)\big) - b\right)_{i\in J}.$$

Definition 5 (Disclosure regularity condition)**.** *We say that the above $b = 0$ efficient disclosure solution satisfies the disclosure regularity condition if* $\det\left(I - D_t\Phi_0^J(t^\star)\right) \neq 0$, *where I is the identity matrix $\mathbb{R}^{|J|\times|J|}$, and $D_t\Phi_0^J(t^\star)$ is the Jacobian of Φ_0^J with respect to t evaluated at $t^\star$.*

This nondegeneracy condition is what allows the baseline cutoff vector $t^\star$ to be continued to nearby values of b by the implicit function theorem.

Lemma B.4. *Suppose the disclosure regularity condition holds. Then there exists $\hat{b} > 0$ such that for every $b \in [0, \hat{b})$ there exists a PBE of the disclosure game with bias b whose receiver payoff is strictly larger than under mandated disclosure (the FED outcome).*

Proof of Lemma B.4. Fix $(\sigma^\star, \alpha^\star, \mu^\star)$ from Lemma B.3 at $b = 0$ and take an outcome-equivalent generalized-cutoff representation $(L^\star, M^\star, H^\star, t^\star)$ as described right before the statement of the proposition. Write $a_i^\star \triangleq a_i(t^\star)$ for $i \in J$ and $a_0^\star \triangleq a_0(t^\star)$.

Step 1: Interior cutoffs and strict sign pattern. Fix $i \in J$. Since $t_i^* \in (0,1)$, sender optimality requires indifference of type $(\theta, s) = (t_i^\star, s_i)$ at $b = 0$:

$$-(t_i^\star - a_i^\star)^2 = -(t_i^\star - a_0^\star)^2 \quad \Rightarrow \quad t_i^\star = \tfrac{1}{2}(a_i^\star + a_0^\star).$$

Moreover, under the generalized cutoff rule, conditional on $m = s_i$ the state is truncated to a nondegenerate tail:

$$a_i^\star = \begin{cases} \mathbb{E}[\theta \mid s_i, \theta \leq t_i^\star] < t_i^\star & \text{if } i \in L^\star, \\ \mathbb{E}[\theta \mid s_i, \theta \geq t_i^\star] > t_i^\star & \text{if } i \in H^\star, \end{cases}$$

(using full support and atomlessness). Combining with $t_i^\star = \frac{1}{2}(a_i^\star + a_0^\star)$ gives the strict inequalities

$$a_i^\star < a_0^\star \text{ for } i \in L^\star, \qquad a_i^\star > a_0^\star \text{ for } i \in H^\star.$$

Since J is finite, define the strictly positive margin

$$\eta \triangleq \min_{i\in J} |a_i^\star - a_0^\star| > 0.$$

By continuity of $(a_i(\cdot), a_0(\cdot))$ in t, there exists a neighborhood $V \subset (0,1)^J$ of $t^\star$ such that for all $t \in V$,

$$a_i(t) \le a_0(t) - \eta/2 \text{ for } i \in L^\star, \qquad a_i(t) \ge a_0(t) + \eta/2 \text{ for } i \in H^\star.$$

Step 2: Smoothness of the indifference map. Fix the partition $(L^\star, M^\star, H^\star)$. For $t \in V$, each on-path message $m = s_i$ and $m = \circ$ occurs with strictly positive probability. Writing posterior means as ratios of truncated integrals and applying the fundamental theorem of calculus yields that $t \mapsto a_i(t)$ and $t \mapsto a_0(t)$ are continuously differentiable on V. Hence $(t,b) \mapsto \Phi_b^J(t)$ is C^1 on $V \times (-\varepsilon, \varepsilon)$ for some $\varepsilon > 0$.

Step 3: Implicit-function continuation of cutoffs. Define $F(t,b) \triangleq t - \Phi_b^J(t)$. At the baseline equilibrium, sender indifference implies $F(t^\star, 0) = 0$. Moreover,

$$D_t F(t^\star, 0) = I - D_t \Phi_0^J(t^\star),$$

which is invertible by the determinant assumption. The Implicit Function Theorem therefore yields $\hat{b}_1 > 0$ and a unique C^1 map $b \mapsto t(b)$ on $(-\hat{b}_1, \hat{b}_1)$ such that $t(0) = t^\star$ and $F(t(b), b) = 0$ for all b in that interval. Shrink $\hat{b}_1$ if necessary so that $t(b) \in V$ for all $b \in [0, \hat{b}_1)$.

Step 4: Constructing a PBE for each small $b \ge 0$. Fix $b \in [0, \hat{b}_1)$. Let the sender use the generalized cutoff rule with partition $(L^\star, M^\star, H^\star)$ and cutoffs $t(b)$: for $i \in L^\star$ disclose s_i iff $\theta \le t_i(b)$, for $i \in H^\star$ disclose s_i iff $\theta \ge t_i(b)$, and for $i \in M^\star$ remain silent. On path, let the receiver play posterior means: $\alpha^b(s_i) = a_i(t(b))$ for $i \in J$ and $\alpha^b(\circ) = a_0(t(b))$. For off-path messages s_i with $i \in M^\star$, set $\alpha^b(s_i) = \alpha^b(\circ)$ and choose any off-path belief $\mu^b(\cdot \mid s_i)$ supported on $\{s = s_i\}$ with mean $\mathbb{E}_{\mu^b}[\theta \mid s_i] = \alpha^b(\circ)$.

Receiver optimality holds because quadratic loss is uniquely minimized by the posterior mean. For sender optimality, fix $i \in J$ and abbreviate $a_i = a_i(t(b))$, $a_0 = a_0(t(b))$. Since $F(t(b), b) = 0$, we have

$$t_i(b) = \tfrac{1}{2}(a_i + a_0) - b,$$

which is exactly the indifference cutoff for a sender with ideal action $\theta + b$ between actions a_i and a_0. By Step 1 and $t(b) \in V$, we retain $a_i < a_0$ for $i \in L^\star$ and $a_i > a_0$ for $i \in H^\star$, hence the sender strictly prefers to disclose on the intended side of the cutoff and to remain silent on the other side. For $i \in M^\star$, disclosure and silence induce the same action by construction, so silence is optimal. Therefore the constructed assessment is a PBE.

Step 5: Payoff dominance over mandated disclosure for small b. Let $U(b)$ be the receiver payoff in the constructed PBE at bias b and let U^{MD} be the receiver payoff under mandated disclosure. Because $t(b) \to t^\star$ as $b \downarrow 0$ and payoffs are continuous in t, we have $U(b) \to U(0)$. By Lemma B.3, $U(0) > U^{MD}$, so there exists $\hat{b} \in (0, \hat{b}_1]$ such that $U(b) > U^{MD}$ for all $b \in [0, \hat{b})$. □

Proposition B.1. *Suppose the sender observes only s, not θ. Then, for every $b \geq 0$, the most informative equilibrium of the game with verifiable evidence is FED.*

Proof of Proposition B.1. Fix $b \geq 0$. Since, the sender does not observe θ, her strategy only depends on s. Define an assessment (σ, α, μ) such that for all $s \in S$ $\sigma(m|s) = \mathbb{1}\{m = s\}$ and $\alpha(s) = \mathbb{E}[\theta|s]$. Let $\alpha(\circ) = 0$. Let $\mu(\cdot|s)$ be given by Bayes' rule for each $s \in S$ and $\mu(\cdot|\circ)$ give probability 1 to $\theta = 0$. Receiver optimality holds by construction. We verify sender optimality. Fix $s \in S$. The sender's interim expected payoff conditional on s is

$$\mathbb{E}\left[(a - \theta - b)^2 \mid s\right] = \mathrm{Var}(\theta|s) + (a - (\mathbb{E}(\theta \mid s) + b))^2 .$$

Thus, the sender prefers the message $m \in \{s, \circ\}$ inducing the action closest to $\mathbb{E}(\theta \mid s) + b$. Sender optimality requires

$$(\mathbb{E}(\theta \mid s) - (\mathbb{E}(\theta \mid s) + b))^2 \leq (0 - (\mathbb{E}(\theta \mid s) + b))^2 \quad \Longleftrightarrow \quad b^2 \leq (b + \mathbb{E}(\theta \mid s))^2,$$

which always holds. Therefore FED is a PBE. To see that it is the most informative one, we simply note that, in any other PBE, the sender's strategy consists of a garbling of s and thus lead to a weakly lower receiver's payoff. □

B.2 Proofs for Section 4

Lemma B.5. *Let* $b^{CT} \triangleq \sup_{t \in (0,1)} \Delta(t)$ *where* $\Delta(t) \triangleq \left\{\frac{1}{2}\Big(\mathbb{E}[\theta \mid \theta \leq t] + \mathbb{E}[\theta \mid \theta \geq t]\Big) - t\right\}$. *Then:*

(1) *If* $b > b^{CT}$*, every PBE of the cheap-talk game is uninformative (babbling).*

(2) *If* $b < b^{CT}$*, there exists a strictly informative PBE in the cheap-talk equilibrium.*

(3) *If* $b = b^{CT}$*, a strictly informative PBE of the cheap-talk game exists if and only if there* $\Delta(t)$ *attains its supremum at some* $t \in (0,1)$.

Proof of Lemma B.5*. Step 1: Necessity for strict informativeness.* Suppose there exists a strictly informative PBE (σ, α, μ). We want to show that b must be smaller than b^{CT}. Let $\Pr_\sigma(m)$ denote the equilibrium probability of message m. Let $\mathcal{A} \triangleq \{\alpha(m) : \Pr_\sigma(m) > 0\}$ be the set of on-path receiver actions. Since the equilibrium is strictly informative $|\mathcal{A}| \geq 2$ and, letting $a_1 \triangleq \min \mathcal{A}$ and $a_2 \triangleq \min(\mathcal{A} \setminus \{a_1\})$, we have $a_1 < a_2$. Let $t \in (0,1)$ be the cutoff such that

$$(a_1 - t - b)^2 = (a_2 - t - b)^2 \quad \Longleftrightarrow \quad b = \frac{1}{2}(a_1 + a_2) - t.$$

Since a_1 is the lowest action and the equilibrium must be partitional due to the structure of the preferences, we must have $a_1 = \mathbb{E}[\theta | \theta \leq t]$. Next, consider the messages used when $\theta \geq t$. All such on-path messages induce actions weakly above a_2, thus $a_2 \leq \mathbb{E}[\theta | \theta \geq t]$. Thus,

$$b = \frac{1}{2}(a_1 + a_2) - t \leq b^{CT}.$$

Hence, any strictly informative PBE must satisfy $b \leq b^{CT}$. In particular, if $b > b^{CT}$ then no strictly informative PBE exists, so every PBE is uninformative (babbling). This proves (1).

Moreover, if $b = b^{CT}$ and a strictly informative PBE exists, then the chain of inequalities above must hold with equality, which forces $\Delta(t) = b^{CT}$ for some $t \in (0,1)$. Thus the supremum in the definition of b^{CT} is attained. This proves the "only if" direction of (3).

Step 2: Sufficiency for strict informativeness. Define $L(t) \triangleq \mathbb{E}[\theta \mid \theta \leq t]$ and $H(t) \triangleq \mathbb{E}[\theta \mid \theta \geq t]$, so $\Delta(t) = \frac{1}{2}\big(L(t) + H(t)\big) - t$. Then, $b^{CT} = \sup_{t \in (0,1)} \Delta(t)$. Note that $\Delta(t)$ is continuous on $(0,1)$. Fix $b < b^{CT}$. By definition of b^{CT}, there exists $\hat{t} \in (0,1)$ such that $b < \Delta(\hat{t})$. Moreover, for t sufficiently close to 1 we have $\Delta(t) < 0$. By continuity of Δ, there exists $t^* \in (\hat{t}, 1)$ such that $\Delta(t^*) = b$. Fix two distinct messages $m_L, m_H \in M$. Consider the sender strategy

$$m(\theta) = \begin{cases} m_L & \text{if } \theta \leq t^*, \\ m_H & \text{if } \theta > t^*. \end{cases}$$

Given this strategy, Bayes' rule implies that the receiver's posterior beliefs satisfy $\mathbb{E}[\theta \mid m_L] = L(t^*)$, and $\mathbb{E}[\theta \mid m_H] = H(t^*)$. Sender incentive compatibility holds by construction. Since $t^* \in (0,1)$, both messages occur with positive probability, and $L(t^*) < H(t^*)$, hence the constructed PBE is strictly informative.

Step 3: The knife-edge case $b = b^{CT}$. If there exists $t \in (0,1)$ with $\Delta(t) = b^{CT}$, the construction in Step 2 (with $b = b^{CT}$) yields a strictly informative PBE, proving the "if" direction of (3). Conversely, if the supremum is not attained on $(0,1)$, Step 1 shows that any strictly informative PBE at $b = b^{CT}$ would imply the existence of $t \in (0,1)$ with $\Delta(t) = b^{CT}$, a contradiction. Hence, every PBE is babbling in that case. □

Lemma B.6. *We have $\frac{\mathbb{E}[\theta]}{2} \leq b^{CT} \leq \frac{1}{2}$. Furthermore, the lower bound is tight (i.e., there exists a prior F such that $b^{CT} = \frac{\mathbb{E}[\theta]}{2}$), and the upper bound is asymptotically tight (i.e., for every $\varepsilon > 0$ there exists a prior F such that $b^{CT} \geq \frac{1}{2} - \varepsilon$).*

Proof of Lemma B.6. First, we prove that $b^{\text{CT}} \leq \frac{1}{2}$. Let $\mu \triangleq \mathbb{E}[\theta]$ and define, for $t \in (0,1)$, $L(t) \triangleq \mathbb{E}[\theta \mid \theta \leq t]$ and $H(t) \triangleq \mathbb{E}[\theta \mid \theta \geq t]$. Let $\Delta(t) = \frac{1}{2}(L(t) + H(t)) - t$, and note that $b^{\text{CT}} = \sup_{t \in (0,1)} \Delta(t)$. For any F, we observe that $L(t) \leq t$ and $H(t) \leq 1$. Thus, $\Delta(t) \leq \frac{1}{2}(t+1) - t = \frac{1-t}{2}$. Since $t \in (0,1)$, we have $\frac{1-t}{2} < \frac{1}{2}$. Taking the supremum over t yields $b^{CT} \leq \frac{1}{2}$.

Next, we show the upper bound is asymptotically tight. Consider a sequence of distributions F_k with densities

$$f_k(\theta) = \frac{ke^{k\theta}}{e^k - 1}, \quad \theta \in [0,1].$$

This distribution concentrates mass near 1. For a fixed $t > 0$, as $k \to \infty$, we have $H_k(t) \to 1$. Fix $\epsilon > 0$ and choose $t = \frac{\epsilon}{2}$. Since $H_k(t) \to 1$, there exists K such that for all $k \geq K$, $H_n\left(\frac{\epsilon}{2}\right) \geq 1 - \epsilon$. Using the fact that $L_k(t) \geq 0$, for such k we have:

$$b_k^{CT} = \sup_{t \in (0,1)} \Delta_k(t) \geq \Delta_k\left(\frac{\epsilon}{2}\right) \geq \frac{1}{2}(0 + 1 - \epsilon) - \frac{\epsilon}{2} = \frac{1}{2} - \epsilon,$$

which proves asymptotic tightness. Notice that the upper bound cannot be attained. Since $\Delta(t) = \frac{1}{2}(L(t) + H(t)) - t \leq \frac{1}{2}(t+1) - t = \frac{1-t}{2} < \frac{1}{2}$ for $t > 0$, the only way the supremum could still be $\frac{1}{2}$ is if $\Delta(t_n) \to \frac{1}{2}$ along some $t_n \downarrow 0$. But as $t \downarrow 0$, $L(t) = \mathbb{E}[\theta \mid \theta \leq t] \to 0$ and $H(t) = \mathbb{E}[\theta \mid \theta \geq t] \to \mathbb{E}[\theta]$, so $\Delta(t) \to \frac{\mathbb{E}[\theta]}{2}$. Since $\mathbb{E}[\theta] < 1$ (atomless F), $\lim_{t \downarrow 0} \Delta(t) = \frac{\mathbb{E}[\theta]}{2} < \frac{1}{2}$, which rules out $\sup_{t \in (0,1)} \Delta(t) = \frac{1}{2}$.

Next, we show that $\frac{\mathbb{E}[\theta]}{2} \leq b^{\text{CT}}$. Note that by continuity $\lim_{t\downarrow 0} L(t) = 0$ and $\lim_{t\downarrow 0} H(t) = \mathbb{E}[\theta]$. Therefore,

$$b^{CT} \geq \lim_{t\downarrow 0} \left(\frac{1}{2}(L(t) + H(t)) - t \right) = \frac{\mathbb{E}[\theta]}{2}.$$

To see that this bound is tight simply observe that $b^{CT} = \frac{\mathbb{E}[\theta]}{2}$ when F is uniform. □

Fix an integer $L \geq 1$. Let $\mathcal{Q}_L$ be the set of all pairs (t, a) where $t = (t_0, \ldots, t_L) \in [0,1]^{L+1}$ with $0 = t_0 \leq t_1 \leq \cdots \leq t_L = 1$, and $a = (a_1, \ldots, a_L) \in [0,1]^L$. Each t induces L (possibly empty) cells: $I_j(t) = [t_{j-1}, t_j)$ if $j < L$ and $I_L(t) = [t_{L-1}, 1]$. Given (t, a), denote the mean-squared error

$$MSE_L(t,a) \triangleq \sum_{j=1}^{L} \int_{I_j(t)} (\theta - a_j)^2 f(\theta) d\theta.$$

Let $MSE_L^\star \triangleq \min_{(t,a)\in\mathcal{Q}_L} MSE_L(t,a)$. A minimizer exists because $\mathcal{Q}_L$ is compact and MSE_L is continuous. Due to the quadratic loss function, a minimizer $(t^\star, a^\star)$ must be such that $a_j^\star = \mathbb{E}[\theta \mid \theta \in I_j(t^\star)]$, whenever $I_j(t^\star)$ is nondegenerate (i.e., $\Pr(\theta \in I_j(t^\star)) > 0$). Moreover note that when a equals the vector of conditional means of the corresponding cells, we can write

$$MSE_L(t,a) = \sum_{j=1}^{L} \Pr(\theta \in I_j(t)) \text{Var}(\theta \mid I_j(t)). \tag{3}$$

Lemma B.7. *Fix $L \geq 1$. The following hold:*

(1) $MSE_{L+1}^\star < MSE_L^\star$.

(2) Every minimizer of MSE_L induces L nonempty cells (equivalently $t_0^\star < t_1^\star < \cdots < t_N^\star$).

(3) For any minimizer $(t^\star, a^\star)$, for each interior boundary $j = 1, \ldots, L-1$, $(t_j^\star - a_j^\star)^2 = (t_j^\star - a_{j+1}^\star)^2$, equivalently (since $a_j^\star < a_{j+1}^\star$ when cells are nonempty), $t_j^\star = \frac{a_j^\star + a_{j+1}^\star}{2}$.

Proof of Lemma B.7. Fix a minimizer $(t^\star, a^\star)$ for $MSE_L^\star$. Because the cells $\{I_j(t^\star)\}_{j=1}^L$ partition $[0,1]$, there exists j such that $I_j(t^\star)$ has strictly positive probability and $\text{Var}(\theta \mid \theta \in I_j(t^\star)) > 0$. Choose a point $u \in int(I_j(t^\star))$ such that both subcells $I_j^L = [t_{j-1}^\star, u)$ and $I_j^R = [u, t_j^\star)$ have positive probability. Let t' be the $(L+1)$-cutoff vector obtained from $t^\star$ by inserting u as an additional boundary inside cell j and leaving all other boundaries unchanged. Let a' be the conditional mean on the corresponding cells. Denote by I_j° the random variable that selects I_j^L if $\theta \in I_j^L$ and I_j^R if $\theta \in I_j^R$. By the law of total variance,

$$\text{Var}(\theta \mid \theta \in I_j(t^\star)) = \mathbb{E}[\text{Var}(\theta \mid \tilde{I}_j) \mid \theta \in I_j(t^\star)] + \text{Var}(\mathbb{E}[\theta \mid \tilde{I}_j] \mid \theta \in I_j(t^\star)).$$

Since $\mathbb{E}[\theta \mid I_j^L] < \mathbb{E}[\theta \mid I_j^R]$, the last variance term is strictly positive, thus

$$\mathrm{Var}(\theta \mid \theta \in I_j(t^\star)) > \mathbb{E}[\mathrm{Var}(\theta \mid \tilde{I}_j) \mid \theta \in I_j(t^\star)].$$

Therefore, using (3), $MSE_L^\star = MSE_L(t^\star, a^\star) > MSE_{L+1}(t', a') \geq MSE_{L+1}^\star$. Additionally, this strict inequality implies that any minimizer $(t^\star, a^\star)$ of $MSE_{L+1}(t, a)$ must induce a partition with $L+1$ non-empty cells. Indeed, if some cell were empty, the same mean squared error could be achieved with only L cells, implying $MSE_L^\star \leq MSE_{L+1}^\star$, which contradicts the strict inequality above.

Next, let $(t^\star, a^\star)$ minimize $MSE_L(t, a)$. Then the induced cells are all nonempty with $0 = t_0^\star < t_1^\star < \cdots < t_N^\star = 1$, and for each j, $a_j^\star = \mathbb{E}[\theta | \theta \in I_j(t^\star)]$. Fix $j \in \{1, \ldots, L-1\}$. Since F is atomless with full support, each conditional mean lies strictly inside its interval, hence $a_j^\star < t_j^\star < a_{j+1}^\star$. Define the loss-difference function $g(\theta) \triangleq (\theta - a_j^\star)^2 - (\theta - a_{j+1}^\star)^2 = (a_{j+1}^\star - a_j^\star)(2\theta - a_j^\star - a_{j+1}^\star)$. Then g is continuous and strictly increasing, and vanishes uniquely at $m_j^\star \triangleq \frac{a_j^\star + a_{j+1}^\star}{2}$. We claim that $t_j^\star = m_j^\star$. Suppose first that $t_j^\star < m_j^\star$. Then by continuity of g, there exists $\varepsilon > 0$ such that $t_j^\star + \varepsilon < m_j^\star$ and $g(\theta) < 0$ for all $\theta \in [t_j^\star, t_j^\star + \varepsilon]$. Define a perturbed cutoff vector t' by setting $t_j' = t_j^\star + \varepsilon$ and leaving all other cutoffs unchanged. Keep the action vector fixed at $a^\star$. Only the assignment of states in $[t_j^\star, t_j^\star + \varepsilon)$ changes. Therefore,

$$MSE_L(t', a^\star) - MSE_L(t^\star, a^\star) = \int_{t_j^\star}^{t_j^\star + \varepsilon} g(\theta)\, f(\theta)\, d\theta.$$

Because F is atomless with full support,

$$\int_{t_j^\star}^{t_j^\star + \varepsilon} f(\theta) d\theta = F(t_j^\star + \varepsilon) - F(t_j^\star) > 0.$$

Since $g(\theta) < 0$ on this interval and is bounded away from zero there, the integral is strictly negative. Thus $MSE_L(t', a^\star) < MSE_L(t^\star, a^\star)$, contradicting optimality of $(t^\star, a^\star)$.

The case $t_j^\star > m_j^\star$ is symmetric: shifting the boundary slightly to the left strictly lowers MSE_K.

Therefore $t_j^\star = m_j^\star$, which is equivalent to $(t_j^\star - a_j^\star)^2 = (t_j^\star - a_{j+1}^\star)^2$. □

Proof of Proposition 4. Suppose evidence is informative.

We begin by proving the second bullet. Let $b \geq \max\{\frac{1}{2}, \bar{b}\}$. By Lemmas B.5 and B.6, if $b \geq \frac{1}{2}$, all equilibria of the cheap-talk game are uninformative. When $b \geq \bar{b}$, Proposition

2 establishes that the efficient equilibrium of the disclosure game is at least as informative as the FED outcome, which is informative since evidence is informative. Therefore, when $b \geq \max\{\frac{1}{2}, \bar{b}\}$, the efficient equilibrium when evidence is unverifiable is less informative than the efficient equilibrium when evidence is verifiable.

We now prove the first bullet. Let $b = 0$. We want to show that the efficient equilibrium outcome in the cheap-talk game is more informative than the efficient equilibrium outcome in the disclosure game. Let $K = N + 1$. Consider the auxiliary minimization problem $\min_{(t,a) \in \mathcal{Q}_K} MSE_K(t, a)$ and let $(t^\star, a^\star)$ be a minimizer. By Lemma B.7 (2), the cells are nonempty, so $t^\star$ is strictly increasing; by Lemma B.7 (3), $t^\star$ satisfies the midpoint/indifference condition $t_j^\star = \frac{1}{2}(a_j^\star + a_{j+1}^\star)$ for each interior boundary. Construct a cheap talk assessment from $(t^\star, a^\star)$ as follows. The sender sends message $m = j$ if $\theta \in I_j(t^\star)$ and the receiver responds with $a_j^\star$. Receiver optimality holds because $a_j^\star$ is the conditional mean on $I_j(t^\star)$. Sender optimality holds because with quadratic loss and ordered actions, the midpoint condition implies the midpoint condition implies that for $\theta \in I_j(t^\star)$, action $a_j^\star$ is the closest induced action to θ. Hence the constructed assessment is a PBE of the cheap talk game at $b = 0$. Its induced mean-squared error equals $MSE_K^\star$ by construction. Moreover, any cheap-talk equilibrium with at most K messages induces at most K actions, its MSE cannot be below $MSE_K^\star$. Thus this PBE is an efficient cheap-talk equilibrium.

Next, fix an arbitrary PBE (σ, α, μ) of the disclosure game at $b = 0$. We show it induces a strictly higher MSE than the cheap-talk equilibrium just constructed. Define the realized receiver action random variable $A \triangleq \alpha(m)$, which must take at most K distinct values. We split into two cases.

Case 1: $\Pr(\mathrm{Var}(A \mid \theta) > 0) > 0$. Let $\mathcal{A}$ be the set of on-path action, $|\mathcal{A}| \leq K$. For each θ, let $\hat{a}(\theta) \in \arg\min_{a \in \mathcal{A}} (a - \theta)^2$. Then the conditional MSE satisfies

$$\mathbb{E}[(A - \theta)^2 \mid \theta] \triangleq \sum_{a \in \mathcal{A}} p_a(\theta)(a - \theta)^2 \geq \min_{a \in \mathcal{A}} (a - \theta)^2 = (\hat{a}(\theta) - \theta)^2,$$

with strict inequality whenever the conditional distribution of A given θ assigns positive probability to an action in $\mathcal{A}$ that is not $\hat{a}(\theta)$. Since F is atomless, ties occur only on an F-null set of θ. Hence the assumption $\Pr(\mathrm{Var}(A \mid \theta) > 0) > 0$ implies strict inequality on a set of θ of positive F-measure. Integrating yields $\mathbb{E}[(A - \theta)^2] > \mathbb{E}[(\hat{a}(\theta) - \theta)^2]$. The selector $\hat{a}(\theta)$ takes values in $\mathcal{A}$ and therefore uses at most K action levels. Thus its MSE is bounded below: $\mathbb{E}[(\hat{a}(\theta) - \theta)^2] \geq MSE_K^\star$. Combining, $\mathbb{E}[(A - \theta)^2] > MSE_K^\star$.

Case 2: $\text{Var}(A \mid \theta) = 0$ *for* F*-almost every* θ. Fix such a θ. Let $p_\circ(\theta)$ be the probability message $\circ$ is sent conditional on θ. If $p_\circ(\theta) > 0$, then on the event $\{m = \circ\}$ the receiver action equals $\alpha(\circ)$. Since A is almost surely constant given θ, we must have $A = \alpha(\circ)$ almost surely given θ. If instead $p_\circ(\theta) = 0$, then the sender discloses some signal with probability one. Since A is almost surely constant given θ, all disclosed signals used with positive probability must induce the same action, so $A = c$ almost surely given θ. Thus, for F-almost every θ, the conditional distribution of A given θ is supported on the set $\{\alpha(\circ), c\}$. Therefore A uses at most two action levels overall, implying $\mathbb{E}[(A-\theta)^2] \geq MSE_2^\star > MSE_K^\star$, since by assumption $N \geq 2$ and thus $K \geq 3$.

We are left to extend the strict ranking to small $b > 0$. Let $MSE^{CT}(b)$ denote the infimum mean-squared error across all PBE outcomes of the cheap-talk game with bias b, and define $MSE^D(b)$ analogously for the disclosure game. From the argument established above for $b = 0$, we have the strict gap $MSE^{CT}(0) < MSE^D(0)$.

Let (σ, a, μ) be any disclosure-game PBE at bias $b \geq 0$, and let φ be the induced law of (θ, s, m). By verifiability, φ satisfies the feasibility constraints of the ancillary problem defined in Step 1 of the proof of Lemma B.3. Therefore, the receiver payoff in this PBE is bounded above by the value $W^\star$ of the ancillary problem. By Step 2 of Lemma B.3, at $b = 0$ this bound is attained by some PBE, so $W^\star$ equals the efficient disclosure equilibrium payoff at $b = 0$. Since receiver payoff equals minus MSE under posterior-mean actions, this implies

$$MSE^D(b) \geq MSE^D(0) = -W^\star \qquad \text{for all } b \geq 0.$$

Let $(t^\star, a^\star)$ be a minimizer of $MSE_K^\star$ used in the $b = 0$ cheap-talk equilibrium construction above. By Lemma B.8 below, there exists $\tilde{b} > 0$ and a cheap-talk equilibrium branch $t(b)$ for $b \in [0, \tilde{b})$ such that

$$\lim_{b \downarrow 0} MSE_K(t(b), a(t(b))) = MSE_K^\star. \tag{4}$$

Since $MSE^{CT}(b)$ is the infimum equilibrium MSE in the cheap-talk game, we have

$$MSE^{CT}(b) \leq MSE_K(t(b), a(t(b))) \qquad \forall b \in [0, \tilde{b}).$$

Let $\Delta := MSE^D(0) - MSE^{CT}(0) > 0$. By (4), there exists $b_0 \in (0, \tilde{b})$ such that for all $b \in [0, b_0)$,

$$MSE_K(t(b), a(t(b))) < MSE^{CT}(0) + \frac{\Delta}{2} = \frac{1}{2}\Big(MSE^{CT}(0) + MSE^D(0)\Big) < MSE^D(0).$$

Combining the inequalities yields

$$MSE^{CT}(b) \le MSE_K(t(b), a(t(b))) < MSE^D(0) \le MSE^D(b).$$

Hence the efficient cheap-talk equilibrium is strictly more informative than the efficient disclosure equilibrium for all $b \in [0, b_0)$. This proves the first bullet.

□

To extend the comparison to a neighborhood of $b = 0$, we continue the K-message cheap-talk equilibrium constructed above. For strictly increasing $t = (t_0, \ldots, t_K)$ with $0 = t_0 < t_1 < \cdots < t_{K-1} < t_K = 1$, let cells $I_j(t) \triangleq [t_{j-1}, t_j)$ and define $a_j(t) \triangleq \mathbb{E}[\theta \mid \theta \in I_j(t)]$. Since t_0 and t_K are fixed, we identify t with its interior coordinates $(t_1, \ldots, t_{K-1})$ when applying the implicit function theorem. Let $\mathcal{T}^\circ \triangleq \{t \in (0,1)^{K-1} : t_1 < t_2 < \cdots < t_{K-1}\}$. Define the cheap-talk indifference map

$$\Psi_b(t) = \Big(\tfrac{1}{2}(a_j(t) + a_{j+1}(t)) - b\Big)_{j \in \{1,\ldots,K-1\}}.$$

Definition 6 (Cheap-talk regularity condition)**.** *Let $K = N + 1 \ge 3$, and let $(t^\star, a^\star)$ be a minimizer of $MSE_K^\star$. We say that the $b = 0$ efficient cheap-talk solution $(t^\star, a^\star)$ satisfies the cheap-talk regularity condition if* $\det\big(I - D_t\Psi_0(t^\star)\big) \neq 0$.

Lemma B.8. *Let $K = N + 1 \ge 3$, and let $(t^\star, a^\star)$ be a minimizer of $MSE_K^\star$. Suppose the cheap-talk regularity condition holds. Then there exist $\tilde{b} > 0$ and a C^1 map $b \mapsto t(b)$ on $[0, \tilde{b})$ with $t(0) = t^\star$ and $t(b) = \Psi_b(t(b))$. For each $b \in [0, \tilde{b})$ the strategy profile induced by $t(b)$ and actions $a_j(t(b))$ is a PBE of the cheap-talk game, and*

$$MSE_K(t(b), a(t(b))) \to MSE_K^\star \quad \text{as } b \downarrow 0.$$

Proof of Lemma B.8. Define $F(t, b) = t - \Psi_b(t)$. By Lemma B.7, $t^\star$ satisfies $F(t^\star, 0) = 0$. For strictly increasing t, each $a_j(t)$ is C^1 in t because it is the ratio of two integrals whose limits depend smoothly on t. Hence $F(t, b)$ is C^1 in a neighborhood of $(t^\star, 0)$. The Jacobian with respect to t is $D_tF(t, b) = I - D_t\Psi_b(t)$. By assumption this matrix is invertible at $(t^\star, 0)$. The implicit function theorem therefore yields $\tilde{b} > 0$ and a unique C^1 function $t(\cdot)$ defined on $(-\tilde{b}, \tilde{b})$ such that $t(0) = t^\star$ and $F(t(b), b) = 0$ for all $|b| < \tilde{b}$. Since $t^\star \in \mathcal{T}^\circ$, which is open in $\mathcal{R}^{K-1}$, continuity implies $t(b) \in \mathcal{T}^\circ$ for all sufficiently small b. Shrink $\tilde{b}$ if necessary so that $t(b) \in \mathcal{T}^\circ$ for all $b \in [0, \tilde{b})$.

We now construct a cheap-talk PBE for each $b \in [0, \tilde{b})$. Fix such b. Given $t(b)$, define the sender strategy $m = j$ iff $\theta \in I_j(t(b))$, and let the receiver play $a_j(t(b))$. Receiver optimality follows from quadratic loss. Sender optimality follows because the indifference condition $(a_j - (\theta + b))^2 = (a_{j+1} - (\theta + b))^2$ is satisfied exactly at $\theta = t_j(b)$.

Finally, since $t(b) \to t^\star$ and the integrand is continuous, $MSE_K(t(b), a(t(b))) \to MSE_K^\star$. □

B.3 Proofs for Section 4.1

Proof of Proposition 5. The proof follows from Lemma B.9, Lemma B.10, Lemma B.11, and Proposition B.2. □

Lemma B.9. *Suppose F is uniform. The cheap-talk game admits a unique equilibrium with n on-path actions if and only if $b < \frac{1}{2n(n-1)}$.*

Proof of Lemma B.9. Follows immediately from the explicit characterization of the maximum possible equilibrium partition size in Section 4 of Crawford and Sobel (1982). □

Lemma B.10. *Assume $b < \frac{1}{12}$ and let F be uniform. The two-action equilibrium of the cheap talk game is strictly less informative than the three-action equilibrium of the cheap talk game.*

Proof of Lemma B.10. Follows immediately from Theorem 3 of Crawford and Sobel (1982). □

Lemma B.11. *Assume evidence is uninformative and F is uniform. If $b \geq \frac{1}{2}$, then every equilibrium of the disclosure game is uninformative.*

Proof of Lemma B.11. Let $b \geq \frac{1}{2}$ and suppose by way of contradiction that there is a PBE $(\sigma^*, \alpha^*, \mu^*)$ of the disclosure game that is informative. If $\Pr_{\sigma^*}(m = \circ) = 0$, the equilibrium is uninformative (since the evidence is uninformative), a contradiction. Henceforth, assume $\Pr_{\sigma^*}(m = \circ) > 0$ and define $a_0 \triangleq \mathbb{E}[\theta \mid m = \circ] \in (0, 1)$. Let $a_i \triangleq \mathbb{E}[\theta \mid m = s_i]$ for each $i \in \{1, \ldots, N\}$. Suppose there exists an on-path i such that $a_i \neq a_0$ (otherwise the equilibrium is uninformative, again a contradiction).

If $a_i < a_0$, disclosure occurs iff $\theta \leq t_i$ with $t_i = \frac{a_i + a_0}{2} - b$. Uniformity implies $a_i = \frac{t_i}{2}$, hence $a_0 = \frac{3}{2} t_i + 2b > 1$, a contradiction. If $a_i > a_0$, disclosure occurs iff $\theta \geq t_i$ with

$t_i = \frac{a_i + a_0}{2} - b$. Uniformity implies $a_i = \frac{1+t_i}{2}$, hence $a_0 - a_i = t_i + 2b - 1 \geq 0$, contradicting $a_i > a_0$. □

Proposition B.2. *Suppose evidence is uninformative, that is, $\pi(s_i \mid \theta) = p_i$ for all $\theta \in [0,1]$, with $p_i > 0$ and $\sum_{i=1}^{N} p_i = 1$, and let F be uniform on $[0,1]$. There exists $b^\circ \in \left(\frac{1}{8}, \frac{1}{4}\right)$ such that:*

- *For all $b \leq b^\circ$, the efficient equilibrium in the disclosure game is as informative as the unique 2-message cheap-talk equilibrium.*
- *For $b \in (b^\circ, 1/2)$, the efficient equilibrium in the disclosure game dominates the efficient cheap-talk equilibrium.*

Proof of Proposition B.2. Because F is uniform on $[0,1]$, $\mathrm{Var}(\theta) = 1/12$, and therefore the receiver's ex-ante payoff is $-\mathbb{E}[\mathrm{Var}(\theta|m)] = -1/12 + \mathrm{Var}(\mathbb{E}[\theta|m])$. Hence ranking equilibrium outcomes by informativeness is equivalent to ranking them by the variance of the posterior mean $\mathrm{Var}(\mathbb{E}[\theta|m])$.

Two-message cheap-talk benchmark for $b < 1/4$. In the game with unverifiable evidence, the unique non-babbling two-message equilibrium (when $b < 1/4$) is characterized by a single cutoff $r \in (0,1)$: regardless of the signal realization, the sender sends m_L for $\theta \in [0,r)$ and m_H for $\theta \in [r,1]$. Receiver optimality gives $a_L = \mathbb{E}[\theta \mid m_L] = r/2$ and $a_H = \mathbb{E}[\theta \mid \theta \geq r] = (1+r)/2$. Sender indifference at $\theta = r$ implies $r + b = \frac{a_L + a_H}{2} = \frac{2r+1}{4}$, that is, $r = 1/2 - 2b$. Define $V^{CT2}(b) := \mathrm{Var}(\mathbb{E}[\theta \mid m^{CT2}])$ for this two-message equilibrium. Since the posterior mean takes the values a_L and a_H with probabilities r and $(1-r)$,

$$V^{CT2}(b) = r(1-r)\,(a_H - a_L)^2 = \frac{1}{16} - b^2.$$

For $b \geq 1/4$ the cheap-talk equilibrium is babbling so the efficient cheap-talk equilibrium has variance 0.

A disclosure equilibrium achieving $V^{CT2}(b)$. There exists an equilibrium of the disclosure game that achieves $V^{CT2}(b)$. Fix the same cutoff $r = 1/2 - 2b$. Consider the following disclosure strategy: for every (θ, s_i), disclose $m = s_i$ if and only if $\theta \geq r$, and send $m = \circ$ otherwise. Since evidence is uninformative, this strategy can be sustained in equilibrium. We omit the straightforward verification. Therefore the efficient disclosure equilibrium achieves at least $V^{CT2}(b)$.

Structure of any disclosure equilibrium. Fix any PBE of the disclosure game. If $\Pr(m = \circ) = 0$, then only evidence messages are observed and, because evidence is uninformative, $\mathbb{E}[\theta|m] = 1/2$ almost surely, so $\text{Var}(\mathbb{E}[\theta \mid m]) = 0$. Hence assume $\Pr_\sigma(m = \circ) > 0$. By Lemma 1, we may restrict attention to generalized cutoff rules. Let $a_0 \triangleq \alpha(\circ)$ and $a_i \triangleq \alpha(s_i)$, and define $L \triangleq \{i : a_i < a_0\}$, $M \triangleq \{i : a_i = a_0\}$, and $H \triangleq \{i : a_i > a_0\}$. For $i \in L$, disclosure occurs iff $\theta \leq t_i$, and since the prior is uniform, $a_i = \mathbb{E}[\theta \mid \theta \leq t_i] = t_i/2$. For $i \in H$, disclosure occurs iff $\theta \geq t_i$, and since the prior is uniform, $a_i = \mathbb{E}[\theta \mid \theta \geq t_i] = (t_i + 1)/2$. Indifference of the cutoff type gives $t_i = (a_i + a_0)/2 - b$. Substituting the expressions for a_i yields

$$t_i = \begin{cases} \frac{2a_0 - 4b}{3} \triangleq t_L & \text{if } i \in L, \\ \frac{1+2a_0-4b}{3} = \frac{1}{3} + t_L \triangleq t_H & \text{if } i \in H. \end{cases}$$

Thus every disclosure equilibrium induces at most three on-path actions:

$$a_L \triangleq \frac{t}{2}, \qquad a_0 \triangleq \frac{3t+4b}{2}, \qquad a_H \triangleq \frac{t}{2} + \frac{2}{3},$$

where $t \triangleq t_L \in (0, 2/3)$.

Three-action disclosure equilibria never beat $V^{CT2}(b)$. Suppose both L and H are nonempty. Let

$$\lambda \triangleq \sum_{i \in L} p_i, \qquad \rho \triangleq \sum_{i \in H} p_i, \qquad \tau \triangleq \sum_{i \in M} p_i = 1 - \lambda - \rho.$$

For fixed t, the on-path probabilities are $\Pr(a_L) = \lambda t$, $\Pr(a_H) = \rho\left(\frac{2}{3} - t\right)$, and $\Pr(a_0) = \lambda(1-t) + \rho\left(t + \frac{1}{3}\right) + \tau$. Bayes' rule after silence gives

$$a_0 = \frac{\lambda(1-t)\frac{1+t}{2} + \rho\left(t + \frac{1}{3}\right)\frac{t+\frac{1}{3}}{2} + \tau\frac{1}{2}}{\lambda(1-t) + \rho\left(t + \frac{1}{3}\right) + \tau}. \tag{5}$$

Since $a_0 = (3t + 4b)/2$ is already fixed, equation (5) is equivalent, after cross-multiplication, to a linear restriction on (λ, ρ, τ). Together with $\lambda + \rho + \tau = 1$ and $\lambda, \rho, \tau \geq 0$, this defines a convex compact feasible set. Moreover, the induced second moment of the posterior mean is

$$\mathbb{E}\left[\left(\mathbb{E}[\theta \mid m]\right)^2\right] = \lambda t\, a_L^2 + \rho\left(\frac{2}{3} - t\right)a_H^2 + \left(\lambda(1-t) + \rho\left(t + \frac{1}{3}\right) + \tau\right)a_0^2$$

is affine in (λ, ρ, τ). Since $\text{Var}(\mathbb{E}[\theta \mid m]) = \mathbb{E}[(\mathbb{E}[\theta \mid m])^2] - (\mathbb{E}[\theta])^2 = \mathbb{E}[(\mathbb{E}[\theta \mid m])^2] - 1/4$, maximizing the variance of the posterior mean is equivalent to maximizing the second moment.

Therefore, the maximal variance, for fixed t, is attained at an extreme point of the feasible set. Since the feasible set is contained in the simplex $\{(\lambda, \rho, \tau) \in \mathbb{R}^3_+ : \lambda + \rho + \tau = 1\}$, at an extreme point at least one of λ, ρ, τ is zero. If $\lambda = 0$ or $\rho = 0$, the equilibrium collapses to the two-action case analyzed below. Hence it suffices to focus on the boundary case $\tau = 0$, i.e. $M = \varnothing$.

Suppose $\tau = 0$. Then $\rho = 1 - \lambda$, and (5) becomes

$$a_0 = \frac{\lambda(1-t)\frac{1+t}{2} + (1-\lambda)\left(t + \frac{1}{3}\right)\frac{t+\frac{1}{3}}{2}}{\lambda(1-t) + (1-\lambda)\left(t + \frac{1}{3}\right)}.$$

Using $a_0 = (3t + 4b)/2$, we obtain

$$\lambda = \frac{(3t+1)(12b+6t-1)}{4(18bt - 6b + 9t^2 - 6t + 2)}.$$

A direct simplification yields

$$\left(\frac{5}{16} - b^2\right) - \mathbb{E}\left[\left(\mathbb{E}[\theta \mid m]\right)^2\right] = \frac{(4b+2t-1)(12b+6t-1)(-18bt+6b+15t^2-10t-2)}{48(18bt-6b+9t^2-6t+2)}.$$

For $t \in (0, 2/3)$ and $b < 1/4$, the denominator is positive. Since $\lambda \in (0,1)$, the formula above implies $12b + 6t - 1 > 0$ and $4b + 2t - 1 < 0$. Finally, $-18bt + 6b + 15t^2 - 10t - 2 \leq -2 + 6b < 0$. Hence the right-hand side is strictly positive, so

$$\mathbb{E}\left[\left(\mathbb{E}[\theta \mid m]\right)^2\right] < \frac{5}{16} - b^2.$$

Since $\mathbb{E}\left[\mathbb{E}[\theta \mid m]\right] = 1/2$, this is equivalent to $\mathrm{Var}(\mathbb{E}[\theta \mid m]) < V^{CT2}(b)$. Therefore no genuine three-action disclosure equilibrium can outperform the two-message cheap-talk benchmark.

Two-action disclosure equilibria. By symmetry it is enough to consider $L = \varnothing$ and $H \neq \varnothing$. Let $\rho \in (0, 1]$ and let $t \in (0,1)$ be the common cutoff for all $i \in H$. Then

$$a_H = \frac{1+t}{2}, \qquad a_0 = \frac{(1-\rho)\frac{1}{2} + \rho\frac{t^2}{2}}{1 - \rho + \rho t} = \frac{1 - \rho + \rho t^2}{2(1 - \rho + \rho t)}.$$

Indifference at $\theta = t$ gives $t = (a_H + a_0)/2 - b$, equivalently

$$f(t) := 2\rho t^2 + \big(4\rho(b-1) + 3\big)t + 2(1-\rho)(2b-1) = 0.$$

If $\rho < 1$, then $f(0) < 0 < f(1)$, so $f(t) = 0$ has a unique root $t = t(b, \rho) \in (0, 1)$. If $\rho = 1$, the roots are 0 and $1/2 - 2b$; the nonzero root gives exactly the 2-message cheap-talk benchmark.

For such an equilibrium, the posterior mean takes the two values a_H and a_0, so

$$V^{D2}(b;\rho) := \operatorname{Var}\big(\mathbb{E}[\theta \mid m]\big) = \rho(1-t)\big(1-\rho(1-t)\big)(a_H - a_0)^2.$$

Using $a_H + a_0 = 2(t+b)$, we get $a_H - a_0 = 1 - t - 2b$, $V^{D2}(b;\rho) = \rho(1-t)\big(1-\rho(1-t)\big)(1-t-2b)^2$. For $\rho \in (0,1)$, the equality $V^{D2}(b;\rho) = V^{CT2}(b)$ is equivalent to

$$Q(b,\rho) := 64\rho b^2 + (108 - 100\rho)b + (25\rho - 27) = 0.$$

Moreover,

$$Q(1/8,\rho) = \frac{27}{2}(\rho - 1) < 0, \qquad Q(1/4,\rho) = 4\rho > 0.$$

Therefore, for each $\rho \in (0,1)$, there is a unique $b(\rho) \in (1/8, 1/4)$ such that $V^{D2}(b(\rho);\rho) = V^{CT2}(b(\rho))$. By uniqueness of the crossing, $b < b(\rho)$ implies $V^{D2}(b;\rho) < V^{CT2}(b)$ and $b > b(\rho)$ implies $V^{D2}(b;\rho) > V^{CT2}(b)$. Thus, for every $\rho \in (0,1)$, the two-action disclosure equilibrium is less informative than, equal to, or more informative than the two-message cheap-talk equilibrium according as $b < b(\rho)$, $b = b(\rho)$, or $b > b(\rho)$.

Now let

$$\mathcal{R}(\pi) := \left\{ \sum_{i \in H} p_i : \ H \subseteq \{1, \ldots, N\} \right\} \subseteq (0,1],$$

and define

$$b^\circ(\pi) := \min\{b(\rho) : \rho \in \mathcal{R}(\pi),\ \rho < 1\}.$$

Because $N \geq 2$ and $p_i > 0$ for all i, the set $\mathcal{R}(\pi) \cap (0,1)$ is nonempty and finite, so

$$b^\circ(\pi) \in (1/8, 1/4).$$

Comparison with cheap talk for $b < 1/2$. If $b \leq b^\circ(\pi)$, we claim that no disclosure equilibrium can yield variance above $V^{CT2}(b)$. If $\Pr(m = \circ) = 0$, we already know $\operatorname{Var}(\mathbb{E}[\theta \mid m]) = 0 < V^{CT2}(b)$. If the equilibrium has three on-path actions, the analysis before gives $\operatorname{Var}(\mathbb{E}[\theta \mid m]) < V^{CT2}(b)$. Finally, if the equilibrium has two on-path actions, it is characterized by some $\rho \in \mathcal{R}(\pi)$, and by the definition of $b^\circ(\pi)$, we have $V^{D2}(b;\rho) \leq V^{CT2}(b)$, with equality only when $\rho = 1$. Since $\rho = 1$ is feasible, the efficient disclosure equilibrium achieves $V^{CT2}(b)$.

If $b \in (b^\circ(\pi), 1/4]$, choose $\rho^* \in \mathcal{R}(\pi)$ such that $b^\circ(\pi) = b(\rho^*)$. Then $V^{D2}(b;\rho^*) > V^{CT2}(b)$. Since $b^\circ(\pi) > 1/8 > 1/12$, the efficient cheap-talk equilibrium on $(b^\circ(\pi), 1/4)$ is exactly the two-message equilibrium, while at $b = 1/4$ cheap talk is babbling. Therefore, for every $b \in (b^\circ(\pi), 1/4]$, the efficient disclosure equilibrium dominates the efficient cheap-talk equilibrium.

Finally, if $b \in (1/4, 1/2)$, then cheap talk is babbling, so the efficient cheap-talk equilibrium has variance 0. On the other hand, for every $\rho \in (0,1)$, there exists a two-action disclosure equilibrium with cutoff $t = t(b,\rho) \in (0,1)$. Moreover, since $f(1-2b) = 1-2b > 0$ and $f(0) < 0$, the unique root satisfies $t < 1-2b$, so $a_H - a_0 = 1 - t - 2b > 0$. Hence $V^{D2}(b;\rho) = \rho(1-t)(1-\rho(1-t))(a_H - a_0)^2 > 0$. Therefore the disclosure game admits a strictly informative two-action equilibrium for every $b \in (1/4, 1/2)$, even though cheap talk is babbling. Combined with Lemma B.11, the threshold $b = 1/2$ is sharp.

□

Proposition B.3. *Assume the evidence structure (S, π) is uninformative and let F be an arbitrary prior distribution. Let $b^{CT} \triangleq \sup_{t\in(0,1)} \left\{ \frac{1}{2}\Big(\mathbb{E}[\theta \mid \theta \le t] + \mathbb{E}[\theta \mid \theta \ge t] \Big) - t \right\}$. We have $\frac{\mathbb{E}[\theta]}{2} \le b^{CT} \le \frac{1}{2}$. Then,*

- *If $b = 0$, the efficient cheap-talk equilibrium outcome dominates the efficient disclosure equilibrium outcome.*
- *Suppose $\mathbb{E}(\theta) \ge \frac{1}{2}$. Then, for all $b \in \big(b^{\mathrm{CT}}, \mathbb{E}(\theta)\big)$, the efficient disclosure equilibrium outcome dominates the efficient cheap-talk equilibrium outcome.*
- *If $b \ge 1$, the efficient equilibrium outcome is uninformative regardless of whether evidence is verifiable.*

Proof of Proposition B.3. The first bullet is proven in Lemma B.12. The second bullet relies on the fact that when F is such that $\mathbb{E}(\theta) \ge \frac{1}{2}$, the interval $\big(b^{\mathrm{CT}}, \mathbb{E}(\theta)\big)$ is non-empty (by Lemma B.6). In this interval, the efficient equilibrium outcome in the cheap talk game is uninformative (Lemma B.5). In contrast, since $b < \mathbb{E}(\theta)$, there exists an informative equilibrium in the disclosure game (Lemma B.1). Finally, the third bullet follows immediately from Lemma B.5, Lemma B.6, and Proposition 3 (first clause), and the fact that evidence is uninformative. □

Lemma B.12. *Assume evidence is uninformative and $b = 0$. Fix $N \geq 2$. The efficient equilibrium outcome in the disclosure game is as informative as a most informative two-message equilibrium in the cheap-talk game, which is in turn less informative than the efficient equilibrium outcome in the cheap talk game.*

Proof of Lemma B.12. Assume the evidence structure is uninformative and $b = 0$. Let $\mu \equiv \mathbb{E}[\theta]$. Since the receiver chooses posterior means under quadratic loss, $\mathbb{E}[u(\theta, \alpha(m))] = -\mathbb{E}[(\alpha(m) - \theta)^2] = -\operatorname{Var}(\theta) + \operatorname{Var}(\mathbb{E}[\theta \mid m])$. Hence ranking equilibrium outcomes by welfare is equivalent to ranking them by $\operatorname{Var}(\mathbb{E}[\theta \mid m])$.

For $u \in (0,1)$, define $L(u) := \mathbb{E}[\theta \mid \theta \leq u]$ and $H(u) := \mathbb{E}[\theta \mid \theta \geq u]$. Let $\Psi(u) := F(u)L(u)^2 + (1 - F(u))H(u)^2$ for $u \in (0,1)$, with endpoint convention $\Psi(0) = \Psi(1) = \mu^2$.

An upper bound for disclosure equilibria. Fix any PBE of the disclosure game. By Lemma 1, without loss of outcome we may assume the sender uses a generalized cutoff rule. Let $Y := \mathbb{E}[\theta \mid m, s]$. Since m is measurable with respect to (m, s), $\mathbb{E}[\theta \mid m] = \mathbb{E}[Y \mid m]$. Therefore, by Jensen's inequality,

$$\mathbb{E}\Big[(\mathbb{E}[\theta \mid m])^2\Big] = \mathbb{E}\Big[(\mathbb{E}[Y \mid m])^2\Big] \leq \mathbb{E}[Y^2].$$

Now fix a signal s_i. Under a generalized cutoff rule, conditional on $s = s_i$, the sender either discloses on a lower tail, discloses on an upper tail, or never discloses. Hence for some $t_i \in [0,1]$,

$$\mathbb{E}[Y^2 \mid s = s_i] = \Psi(t_i).$$

Indeed, if disclosure occurs on the lower tail, then Y takes values $L(t_i)$ and $H(t_i)$ with probabilities $F(t_i)$ and $1 - F(t_i)$; if disclosure occurs on the upper tail, the same two values arise with the probabilities swapped; if the signal is never disclosed, then $Y = \mu$ a.s., which equals $\Psi(0) = \Psi(1) = \mu^2$.

Because evidence is uninformative, $\Pr(s = s_i) = p_i$, so

$$\mathbb{E}[Y^2] = \sum_{i=1}^{N} p_i \, \Psi(t_i) \leq \max_{u \in [0,1]} \Psi(u).$$

Therefore every disclosure equilibrium satisfies

$$\operatorname{Var}(\mathbb{E}[\theta \mid m]) = \mathbb{E}\Big[(\mathbb{E}[\theta \mid m])^2\Big] - \mu^2 \leq \max_{u \in [0,1]} \big(\Psi(u) - \mu^2\big). \tag{6}$$

Attainment by a two-message cheap-talk equilibrium and by a disclosure equilibrium. Let $u^* \in \arg\max_{u \in [0,1]} \Psi(u)$. Consider the two-cell partition $[0, u)$ and $[u, 1]$. Its mean-squared error is

$$MSE_2(u) = \int_0^u (\theta - L(u))^2 f(\theta)\, d\theta + \int_u^1 (\theta - H(u))^2 f(\theta)\, d\theta = \mathbb{E}[\theta^2] - \Psi(u).$$

Hence maximizing $\Psi(u)$ is equivalent to minimizing $MSE_2(u)$. By Lemma B.7(2)–(3), any minimizer of MSE_2 is interior and satisfies the midpoint condition $u^* = \frac{L(u^*)+H(u^*)}{2}$. Thus the cheap-talk strategy that sends m_L for $\theta < u^*$ and m_H for $\theta \geq u^*$ is a two-message cheap-talk equilibrium, and its informativeness is $\Psi(u^*) - \mu^2$.

Now construct a disclosure equilibrium as follows: for every $i \in \{1, \ldots, N\}$, type (θ, s_i) discloses s_i iff $\theta \leq u^*$, and otherwise sends $\circ$. Because evidence is uninformative, every disclosed message s_i induces posterior mean $\mathbb{E}[\theta \mid m = s_i] = L(u^*)$, while silence induces posterior mean $\mathbb{E}[\theta \mid m = \circ] = H(u^*)$. The cutoff type (u^*, s_i) is indifferent because $u^* = \frac{L(u^*)+H(u^*)}{2}$. Hence this is a PBE of the disclosure game, and it induces exactly the same distribution of posterior means as the two-message cheap-talk equilibrium. Combining this with (6), the efficient disclosure equilibrium has exactly the same informativeness as a most informative two-message cheap-talk equilibrium.

Efficient cheap talk is strictly more informative. In the unverifiable-evidence game, the message space is $M = S \cup \{\circ\}$, so the game is outcome-equivalent to a standard cheap-talk game with $K = N + 1 \geq 3$ messages. At $b = 0$, the efficient cheap-talk equilibrium is obtained by minimizing MSE_K. By Lemma B.7(1), $MSE_K^* < MSE_2^*$. Equivalently,

$$\mathrm{Var}(\mathbb{E}[\theta \mid m^{CT,K}]) > \mathrm{Var}(\mathbb{E}[\theta \mid m^{CT,2}]).$$

Since the efficient disclosure equilibrium has the same informativeness as a most informative two-message cheap-talk equilibrium, it is strictly less informative than the efficient cheap-talk equilibrium. □

B.4 Proofs for Section 5

Lemma B.13 (Blackwell monotonicity at $b = 0$)**.** *Consider two evidence structures (S, π) and (S, π'), where S, S' are finite. Assume $\pi \succeq \pi'$, i.e., there exists a garbling kernel $\kappa : S \to \Delta(S')$ such that*

$$\pi'(s' \mid \theta) = \sum_{s \in S} \kappa(s' \mid s)\, \pi(s \mid \theta) \qquad \forall (\theta, s') \in \Theta \times S'.$$

If $b = 0$, then the efficient disclosure equilibrium under (S, π) is weakly more informative than the efficient disclosure equilibrium under (S', π').

Proof of Lemma B.13. For any finite alphabet S, let the message space be $M := S \cup \{\circ\}$. A verifiable disclosure policy under an experiment (S, π) is a measurable kernel $\sigma : \Theta \times S \to \Delta(M)$ such that $\sigma(m \mid \theta, s) = 0$ for all $m \notin \{s, \circ\}$. A (pure) receiver action rule is a map $\alpha : M \to [0, 1]$. At $b = 0$, sender and receiver have the same objective. Thus, for a given experiment (S, π), we may evaluate any pair (σ, α) by the common ex ante payoff

$$U(\sigma, \alpha; S, \pi) := -\int_\Theta \sum_{s \in S} \pi(s \mid \theta) \sum_{m \in \{s, \circ\}} \sigma(m \mid \theta, s)(\alpha(m) - \theta)^2 dF(\theta)$$

and define the ancillary value is

$$W^*(S, \pi) := \sup_\sigma \sup_\alpha U(\sigma, \alpha; S, \pi).$$

Because $b = 0$, Lemma B.3 implies that the payoff of an efficient disclosure equilibrium under a given experiment coincides with the corresponding ancillary value W^*. Therefore it suffices to show that that the ancillary value is monotone in the Blackwell order.

Step 1: Deterministic garblings cannot increase W^*. Suppose first that the garbling is deterministic, given by a map $g : S \to S'$. Let π^g denote the induced coarse experiment: $\pi^g(s' \mid \theta) := \sum_{s \in S:\, g(s) = s'} \pi(s \mid \theta)$. We claim that

$$W^*(S, \pi) \geq W^*(S', \pi^g).$$

Take any feasible disclosure policy σ' and action rule α' under the coarse experiment (S', π^g). We construct a feasible pair (σ, α) under the fine experiment (S, π) by

$$\sigma(s \mid \theta, s) := \sigma'(g(s) \mid \theta, g(s)), \qquad \sigma(\circ \mid \theta, s) := \sigma'(\circ \mid \theta, g(s)),$$

and

$$\alpha(s) := \alpha'(g(s)), \qquad \alpha(\circ) := \alpha'(\circ).$$

The idea is that, after observing the fine signal s, the sender under (S, π) behaves exactly as the coarse policy would behave after observing the unique coarse signal $g(s)$. The receiver, in turn, assigns to the disclosed fine signal s the same action that the coarse receiver would assign to its image $g(s)$.

A direct substitution into U shows that $U(\sigma, \alpha; S, \pi) = U(\sigma', \alpha'; S', \pi^g)$. Since (σ', α') was arbitrary, it follows that $W^*(S, \pi) \geq W^*(S', \pi^g)$.

It is natural to ask whether the above argument directly extends to stochastic garblings. The answer is no. To see this, suppose π' is obtained from π by a stochastic garbling κ. A natural idea is to let the sender, after observing s, privately simulate a coarse signal s' according to $\kappa(\cdot \mid s)$ and then imitate the coarse policy corresponding to s'. However, this does not yield a valid imitation argument. In the deterministic case, each fine signal s determines a unique coarse signal $g(s)$, so one can define a single receiver action $\alpha(s) = \alpha'(g(s))$. Under a stochastic garbling, by contrast, the same realized fine signal s may lead to different simulated coarse labels s' on different private random draws. Then one observed fine message s would have to carry several different coarse meanings, but the receiver in the fine game sees only the disclosed fine message s, not the sender's hidden simulated label s'. Hence there is no direct analogue of the deterministic construction above.

For this reason, we use a different argument.

Step 2: Convexity of the ancillary value. Fix a finite alphabet S. For any fixed pair (σ, α), the map $\pi \mapsto U(\sigma, \alpha; S, \pi)$ is linear in π. Hence $\pi \mapsto W^*(S, \pi)$, being the pointwise supremum of linear functionals, is convex.

Step 3: Any stochastic garbling is a mixture of deterministic garblings. Let $\kappa : S \to \Delta(S')$ be a (possibly stochastic) garbling kernel. Let $\mathcal{G} := \{g : S \to S'\}$ be the finite set of deterministic maps. For each $g \in \mathcal{G}$, define the weight $\omega_g := \prod_{s \in S} \kappa(g(s) \mid s)$. Then $\omega_g \geq 0$ and $\sum_{g \in \mathcal{G}} \omega_g = 1$, and $\pi' = \sum_{g \in \mathcal{G}} \omega_g \, \pi^g$. Thus π' is a convex combination of deterministic coarsenings of π.

Step 4: Conclude Blackwell monotonicity. Using Step 2 with alphabet S' and then Step 1,

$$W^*(S', \pi') = W^*\Big(S', \sum_{g \in \mathcal{G}} \omega_g \, \pi^g\Big) \leq \sum_{g \in \mathcal{G}} \omega_g \, W^*(S', \pi^g) \leq \sum_{g \in \mathcal{G}} \omega_g \, W^*(S, \pi) = W^*(S, \pi).$$

Thus the ancillary value is weakly higher under (S, π) than under (S', π') whenever $\pi \succeq \pi'$.

□

Denote by $\overline{V}^{CT2}(b)$ the maximal informativeness over all cheap-talk equilibrium outcomes with at most two on-path messages under the uniform prior. For $b < \frac{1}{4}$, the unique informative two-message cheap-talk equilibrium is characterized by cutoff $r(b) = \frac{1}{2} - 2b$, and $\overline{V}^{CT2}(b) = \frac{1}{16} - b^2$. For $b \geq \frac{1}{4}$, every cheap-talk equilibrium is babbling, so $\overline{V}^{CT2}(b) = 0$.

Lemma B.14. *Fix $b \geq 0$. Let $\pi_{\delta,\varepsilon}$ be as defined in (1), and $I^*(\pi)$ be the supremum of equilibrium informativeness over all PBE of the disclosure game under π. Then $\limsup_{(\delta,\varepsilon)\to(0,0)} I^*(\pi_{\delta,\varepsilon}) \leq \overline{V}^{CT2}(b)$.*

Proof of Lemma B.14. Take any sequence $(\delta_n, \varepsilon_n) \to (0,0)$. For each n, choose a PBE e_n under $\pi_{\delta_n,\varepsilon_n}$ such that $\text{Var}_{e_n}(\mathbb{E}[\theta|m]) \geq I^*(\pi_{\delta_n,\varepsilon_n}) - \frac{1}{n}$. Let $I_n \triangleq I^*(\pi_{\delta_n,\varepsilon_n})$, and $V_n \triangleq \text{Var}_{e_n}(\mathbb{E}[\theta|m])$. Since $V_n \leq I_n$ by definition of I_n, we have $0 \leq I_n - V_n \leq \frac{1}{n}$ and therefore $\limsup_{n\to\infty} I_n = \limsup_{n\to\infty} V_n$. It suffices to show $\limsup_{n\to\infty} V_n \leq \overline{V}^{CT2}(b)$. By Lemma 1, we may take e_n to be induced by a generalized cutoff strategy. In particular, after observing s_1, either s_1 is never disclosed, or there exists a cutoff $t_{1,n} \in [0,1]$ and a tail choice (lower or upper) such that s_1 is disclosed exactly on that tail.

Passing to a subsequence (n_k) if necessary, assume one of these possibilities is fixed along (n_k). If s_1 is never disclosed along (n_k), then $\text{Pr}_{e_{n_k}}(m = s_1) = 0$, and $\text{Pr}_{e_{n_k}}(m = s_2) \leq \text{Pr}_{\pi_{\delta_{n_k},\varepsilon_{n_k}}}(s_2) \to 0$. Hence $V_{n_k} \to 0$. Thus it remains to consider the case in which s_1 is disclosed on a fixed tail along (n_k). Since $t_{1,n_k} \in [0,1]$, after passing to a further subsequence if needed, assume $t_{1,n_k} \to t_1 \in [0,1]$. If $t_1 \in \{0,1\}$, then one of the messages $m = s_1$ or $m = \circ$ has probability tending to zero. Indeed, if disclosure of s_1 is on the lower tail, then $\text{Pr}_{e_{n_k}}(m = s_1) \leq t_{1,n_k} \to 0$ when $t_1 = 0$, while $\text{Pr}_{e_{n_k}}(m = \circ) \leq 1 - t_{1,n_k} + \text{Pr}_{\pi_{\delta_{n_k},\varepsilon_{n_k}}}(s_2) \to 0$ when $t_1 = 1$. If disclosure of s_1 is on the upper tail, the analogous bounds are $\text{Pr}_{e_{n_k}}(m = \circ) \leq t_{1,n_k} + \text{Pr}_{\pi_{\delta_{n_k},\varepsilon_{n_k}}}(s_2) \to 0$ when $t_1 = 0$ and $\text{Pr}_{e_{n_k}}(m = s_1) \leq 1 - t_{1,n_k} \to 0$ when $t_1 = 1$. Since also $\text{Pr}_{e_{n_k}}(m = s_2) \to 0$, it follows that $V_{n_k} \to 0$.

Now suppose $t_1 \in (0,1)$. Then for all large k, both $m = s_1$ and $m = \circ$ are on path. Let $a_{1,n_k} \triangleq \mathbb{E}_{e_{n_k}}[\theta \mid m = s_1]$, and $a_{0,n_k} \triangleq \mathbb{E}_{e_{n_k}}[\theta \mid m = \circ]$. We claim that along this subsequence, $\{a_{1,n_k}, a_{0,n_k}\} \to \left\{\frac{t_1}{2}, \frac{1+t_1}{2}\right\}$. To see this, note first that $\text{Pr}_{\pi_{\delta_n,\varepsilon_n}}(s_2) \to 0$. Hence $\text{Pr}_{e_n}(m = s_2) \leq \text{Pr}_{\pi_{\delta_n,\varepsilon_n}}(s_2) \to 0$. Next, disclosure of s_1 can only occur when s_1 is observed. Since $\pi_{\delta_n,\varepsilon_n}(s_1|\theta) \to 1$ for every fixed $\theta < 1$ and the region $\{\theta \geq 1 - \delta_n\}$ has vanishing mass, Bayes' rule implies that conditioning on $m = s_1$ becomes asymptotically equivalent to conditioning on θ lying in the disclosure tail for s_1 (and similarly conditioning on $m = \circ$ becomes asymptotically equivalent to conditioning on θ lying in the complementary tail). Concretely, if disclosure of s_1 is on the lower tail, then $a_{1,n_k} \to \mathbb{E}[\theta \mid \theta \leq t_1] = \frac{t_1}{2}$ and

$a_{0,n_k} \to \mathbb{E}[\theta \mid \theta > t_1] = \frac{1+t_1}{2}$; if disclosure is on the upper tail the two limits are swapped. Sender optimality at the cutoff implies

$$t_{1,n_k} = \Pi_{[0,1]}\Big(\frac{a_{1,n_k} + a_{0,n_k}}{2} - b\Big) \to \Pi_{[0,1]}\Big(\frac{1+2t_1}{4} - b\Big) = t_1,$$

where $\Pi_{[0,1]}(x) \triangleq \max\{0, \min\{1, x\}\}$ denotes projection onto $[0,1]$. This fixed-point equation in the last equality has the unique solution $t_1 = \max\{\frac{1}{2} - 2b, 0\}$. Since we are in the case $t_1 \in (0,1)$, it follows that $t_1 = \frac{1}{2} - 2b$. Now note that the contribution of $m = s_2$ to the variance V_{n_k} is asymptotically negligible. Moreover, we have $\{a_{1,n_k}, a_{0,n_k}\} \to \{\frac{t_1}{2}, \frac{1+t_1}{2}\}$ and the probability that $m = s_1$ converges to the mass of the disclosure tail, which is either t_1 or $1 - t_1$ depending on the tail choice. Since V_{n_k} is invariant to swapping the labels of s_1 and $\circ$, it follows that

$$\limsup_{k\to\infty} V_{n_k} \le \frac{t_1(1-t_1)}{4} = \max\Big\{\frac{1}{16} - b^2, 0\Big\} = \overline{V}^{CT2}(b).$$

We have shown that every subsequential limit of (V_n) is at most $\overline{V}^{CT2}(b)$. Therefore

$$\limsup_{n\to\infty} V_n \le \overline{V}^{CT2}(b).$$

Since $\limsup_{n\to\infty} I_n = \limsup_{n\to\infty} V_n$, the claim follows. □

Lemma B.14 implies that for any $\eta > 0$ there exist $\bar{\delta} > 0$ and $\bar{\varepsilon} > 0$ such that for all $\delta \in (0, \bar{\delta})$ and $\varepsilon \in (0, \bar{\varepsilon})$, the informativeness of the most informative PBE under $\pi_{\delta,\varepsilon}$ is at most $\overline{V}^{CT2}(b) + \eta$. In particular, as $(\delta, \varepsilon) \to (0,0)$, the efficient equilibrium informativeness cannot exceed $\overline{V}^{CT2}(b)$ in the limit.

C Online Appendix

C.1 General Message Structure

Fix an evidence structure (S, π), where $S = \{s_1, \ldots, s_N\}$ is a finite set of possible signal realizations, and the map $\pi : \Theta \to \Delta(S)$ specifies the conditional distribution of signals given the state. Let $M \triangleq S \cup \{\circ\}$ be the message space.

A *message structure* is a correspondence

$$M(\cdot) : S \rightrightarrows M$$

that assigns to each signal realization $s_i \in S$ a set of feasible messages $M(s_i) \subseteq M$ such that

$$\{s_i, \circ\} \subseteq M(s_i) \subseteq M.$$

The restriction $\{s_i, \circ\} \subseteq M(s_i)$ reflects two basic features of disclosure environments. First, the sender can always choose to remain silent. Second, if the sender discloses evidence, the realized signal itself must be a feasible message. These requirements ensure that the sender cannot be forced to misreport the evidence she observes and always retains the option of withholding it.

This formulation captures how communication is constrained conditional on the realized evidence. By varying the feasible message sets $M(s_i)$, the model allows different degrees of communication flexibility and credibility. Smaller sets $M(s_i)$ impose tighter links between messages and realized evidence, leading to more credible but less flexible communication. Larger sets $M(s_i)$ allow the sender greater flexibility in how she describes the realized evidence, but at the cost of weaker credibility.

Two benchmark cases considered in main text arise as special cases:

Verifiable evidence: $M^V(s_i) = \{s_i, \circ\}$. A sender who observes s_i can either disclose that signal or remain silent. This form of "all-or-nothing" message structure captures the idea of stating "the truth, the whole truth and nothing but the truth."

Unverifiable evidence: $M^U(s_i) = M$. All messages are feasible regardless of the realized signal, so the sender can freely choose any message.

The benchmark cases above correspond to the most restrictive and the most permissive message structures, respectively. More generally, message structures can be compared according to how many messages they allow the sender to send after each realized signal.

Definition 7 (Richer messages)**.** *Let $M(\cdot)$ and $M'(\cdot)$ be two message structures on the same evidence structure (S, π). We say that $M'(\cdot)$ is* richer *than $M(\cdot)$ if*

$$M(s_i) \subseteq M'(s_i) \qquad \text{for every } s_i \in S.$$

If $M'(\cdot)$ is richer than $M(\cdot)$, then after every realized signal the sender has a weakly larger set of feasible messages under $M'(\cdot)$ than under $M(\cdot)$. In particular, the set of feasible deviations under $M(\cdot)$ is a subset of those under $M'(\cdot)$. This observation implies a monotonicity property: if a candidate assessment fails to be an equilibrium under the smaller message structure, it also fails to be an equilibrium under any richer message structure, because every deviation feasible under the former remains feasible under the latter.

Lemma C.1 (Equilibrium monotonicity under richer messages)**.** *Let $M(\cdot)$ and $M'(\cdot)$ be two message structures on the same evidence structure (S, π) such that $M'(\cdot)$ is richer than $M(\cdot)$. Consider an assessment (σ, α, μ) whose support uses only messages that are feasible under $M(\cdot)$; that is, $\operatorname{supp}(\sigma(\cdot \mid \theta, s_i)) \subseteq M(s_i)$ for all (θ, s_i). If (σ, α, μ) is a PBE under $M'(\cdot)$, then it is also a PBE under $M(\cdot)$. Equivalently, if (σ, α, μ) is not a PBE under $M(\cdot)$, then it is not a PBE under $M'(\cdot)$.*

Proof. Fix such an assessment (σ, α, μ). Because $M(s_i) \subseteq M'(s_i)$ for every i, every deviation feasible under $M(\cdot)$ is also feasible under $M'(\cdot)$.

Suppose (σ, α, μ) is a PBE under $M'(\cdot)$. Then no profitable deviation exists for any type (θ, s_i) among the larger set of messages $M'(s_i)$. Since $M(s_i) \subseteq M'(s_i)$, no profitable deviation can exist among the smaller set $M(s_i)$ either. Hence sender sequential rationality holds under $M(\cdot)$. Receiver sequential rationality is unchanged because the on-path messages and beliefs remain the same. Therefore (σ, α, μ) is also a PBE under $M(\cdot)$.

The equivalent statement follows by contraposition. □

C.2 General Analysis of FED at $b = 0$

C.2.1 Existence Characterization

Fix $b = 0$. Let the set of signal realizations be $S = \{s_1, \ldots, s_N\}$. For each signal s_i, let $F_i := F(\cdot|s_i)$ and $a_i := \mathbb{E}[\theta|s_i]$, and let $T_i := \operatorname{supp}(F_i)$, $\underline{\theta}_i := \inf T_i$, and $\overline{\theta}_i := \sup T_i$.

Throughout this subsection, we assume that F_i is well-defined for every i, i.e., $\Pr(s_i) > 0$. Moreover, we focus on the minimal verifiable message structure: $M(s_i) = \{s_i, \circ\}$.

Consider a candidate full-evidence-disclosure (FED) profile: every sender type (θ, s_i) discloses the realized signal s_i and the receiver responds with the action a_i, and after silence the receiver chooses some off-path action $a_0 \in [0, 1]$. We characterize when such a profile can be sustained in equilibrium.

Lemma C.2 (Existence characterization)**.** *An FED equilibrium exists at $b = 0$ if and only if there exists $a_0 \in [0, 1]$ such that*

$$a_0 \in \bigcap_{i=1}^{N} \Big([0, \ell_i] \cup \{a_i\} \cup [r_i, 1]\Big). \tag{7}$$

where $\ell_i := 2\underline{\theta}_i - a_i$, $r_i := 2\overline{\theta}_i - a_i$, with the convention that $[x, y] = \emptyset$ when $y < x$.

Proof. Fix a candidate FED profile with off-path action $a_0 \in [0, 1]$. Since evidence is verifiable, under a FED profile the only deviation available to a type (θ, s_i) is from $m = s_i$ to $m = \circ$. At $b = 0$, type (θ, s_i) weakly prefers disclosure to silence iff $-(a_i - \theta)^2 \geq -(a_0 - \theta)^2$, which can be rearranged as $(a_0 - a_i)(a_0 + a_i - 2\theta) \geq 0$. Therefore, if $a_i < a_0$, disclosure is optimal iff $\theta \leq \frac{a_0 + a_i}{2}$ for all $\theta \in T_i$, which is equivalent to $a_0 \geq 2\overline{\theta}_i - a_i = r_i$; if $a_i > a_0$, disclosure is optimal iff $\theta \geq \frac{a_0 + a_i}{2}$ for all $\theta \in T_i$, which is equivalent to $a_0 \leq 2\underline{\theta}_i - a_i = \ell_i$; if $a_i = a_0$, every type in T_i is indifferent between disclosure and silence. Therefore, for each i, the FED incentive constraints hold if and only if

$$a_0 \in [0, \ell_i] \cup \{a_i\} \cup [r_i, 1].$$

Intersecting across $i = 1, \ldots, N$ gives the stated condition (7).

It remains to show sufficiency. Suppose there exists $a_0 \in [0, 1]$ satisfying (7). Consider the candidate FED profile above. By construction, the receiver is sequentially rational after each on-path message s_i, since $\alpha(s_i) = \mathbb{E}[\theta \mid s_i] = a_i$. After the off-path message $\circ$, choose any belief with mean a_0, so that the receiver's best reply is $\alpha(\circ) = a_0$. The conditions above imply that every type (θ, s_i) weakly prefers disclosure to silence. Hence the profile is a PBE that induces the FED outcome. ◻

The following remarks collect two important special cases.

Remark C.1 (Full-support evidence)**.** *Suppose that $T_i = [0,1]$ for every i, so each posterior F_i has full support. If the posterior means $a_1, \dots, a_N$ are not all equal, then no FED equilibrium exists at $b = 0$.*

Proof. If $T_i = [0,1]$, then $\underline{\theta}_i = 0$ and $\overline{\theta}_i = 1$, so $\ell_i = -a_i$ and $r_i = 2 - a_i$. Hence, inside $[0,1]$,

$$[0, \ell_i] \cup \{a_i\} \cup [r_i, 1] = \{a_i\}.$$

By Lemma C.2, a FED equilibrium exists only if there is $a_0 \in [0,1]$ such that $a_0 \in \bigcap_{i=1}^{N} \{a_i\}$. This is possible if and only if $a_1 = \cdots = a_N$. Therefore, if the a_i are not all equal, FED cannot exist at $b = 0$. □

Remark C.2 (Deterministic evidence)**.** *Suppose the evidence structure is deterministic. That is, there exist cutoffs $0 = c_0 < c_1 < \cdots < c_N = 1$ such that $T_i = [c_{i-1}, c_i]$ for all $i = 1, \dots, N$. Then a FED equilibrium exists at $b = 0$.*

Proof. Under deterministic evidence we have $\underline{\theta}_i = c_{i-1}$ and $\overline{\theta}_i = c_i$, so $\ell_i = 2c_{i-1} - a_i$ and $r_i = 2c_i - a_i$.

Consider the candidate off-path action $a_0 = a_k$. We show that there exists k such that $a_k \in [0, \ell_i] \cup \{a_i\} \cup [r_i, 1]$ for all i. Define $L := \{k : c_i \le (a_i + a_k)/2 \text{ for all } i < k\}$. Since the condition is vacuous for $k = 1$, L is nonempty; let $k^* := \max L$. Then for all $i < k^*$ we have $c_i \le (a_i + a_{k^*})/2$, equivalently $a_{k^*} \ge r_i$. So k^* satisfies all the required constraints on the left.

It remains to verify that k^* also satisfies all the required constraints on the right, namely that for every $i > k^*$, $a_{k^*} \le \ell_i = 2c_{i-1} - a_i$. Suppose not. Then there exists $i > k^*$ such that $a_i > 2c_{i-1} - a_{k^*}$. We claim that $i \in L$, contradicting the maximality of k^*. Indeed, fix any $j < i$. We show that $c_j \le (a_j + a_i)/2$. If $j < k^*$, this follows from $k^* \in L$, since $c_j \le (a_j + a_{k^*})/2 < (a_j + a_i)/2$, where the strict inequality uses $a_i > a_{k^*}$. If $k^* \le j < i$, then $c_j \le c_{i-1}$ and $a_j \ge a_{k^*}$, so $2c_j - a_j \le 2c_{i-1} - a_{k^*} < a_i$. Thus $c_j \le (a_j + a_i)/2$ for all $j < i$, so $i \in L$, a contradiction. Hence $a_{k^*} \le \ell_i$ for all $i > k^*$.

Therefore, for $i < k^*$ we have $a_{k^*} \in [r_i, 1]$, for $i > k^*$ we have $a_{k^*} \in [0, \ell_i]$, and for $i = k^*$ we trivially have $a_{k^*} \in \{a_i\}$. Hence $a_{k^*} \in [0, \ell_i] \cup \{a_i\} \cup [r_i, 1]$ for all i. By Lemma C.2, a FED equilibrium exists at $b = 0$. □

Remark C.3 (Arbitrary signal spaces)**.** *The three results above extend to arbitrary evidence structures with possibly infinitely many signal realizations. Let S be an arbitrary measurable* *signal space with signal distribution λ, and for λ-almost every signal s let $F_s := F(\cdot|s)$ denote the posterior distribution. Define*

$$a(s) := \int \theta \, dF_s(\theta), \qquad T(s) := \mathrm{supp}(F_s), \qquad \underline{\theta}(s) := \inf T(s), \qquad \overline{\theta}(s) := \sup T(s),$$

and

$$\ell(s) := 2\underline{\theta}(s) - a(s), \qquad r(s) := 2\overline{\theta}(s) - a(s).$$

The characterization in Lemma C.2 continues to hold with the index i replaced by the signal s: a FED equilibrium exists at $b = 0$ if and only if there exists $a_0 \in [0, 1]$ such that

$$a_0 \in [0, \ell(s)] \cup \{a(s)\} \cup [r(s), 1] \qquad \textit{for } \lambda\textit{-a.e. } s.$$

In particular, if $\mathrm{supp}(F_s) = [0, 1]$ for λ-almost every s, then the admissible set above reduces to $\{a(s)\}$ for almost every s, so a FED equilibrium can exist only if $a(s)$ is λ-a.s. constant. Thus the full-support nonexistence result remains valid for arbitrary evidence structures. The deterministic-evidence result also extends directly to deterministic finite partitions of the state space.

C.2.2 Genericity of Nonexistence of FED at $b = 0$

Fix $N \geq 2$, a prior $F \in \Delta([0, 1])$ with full support, and strictly positive signal probabilities $p = (p_1, \ldots, p_N)$, $p_i > 0$, $\sum_{i=1}^N p_i = 1$. We represent an experiment by its induced posterior laws $E = (F_1, \ldots, F_N) \in \Delta([0, 1])^N$ satisfying Bayes plausibility $\sum_{i=1}^N p_i F_i = F$. Let

$$\mathcal{E}(F, p) := \left\{ (F_1, \ldots, F_N) \in \Delta([0, 1])^N : \sum_{i=1}^N p_i F_i = F \right\},$$

endowed with the product weak topology.

For $E = (F_1, \ldots, F_N) \in \mathcal{E}(F, p)$, write $a_i(E) := \int \theta \, dF_i(\theta)$ for $i = 1, \ldots, N$. Let

$$\mathcal{E}^{\text{info}}(F, p) := \{E \in \mathcal{E}(F, p) : a_1(E), \ldots, a_N(E) \text{ are not all equal}\}.$$

This restriction rules out the degenerate case in which all posterior means coincide, in which case FED may exist even under full support (see Remark C.1).

At $b = 0$, let

$$\mathcal{N} := \left\{ E \in \mathcal{E}^{\text{info}}(F, p) : \text{no FED equilibrium exists at } b = 0 \right\}, \quad \mathcal{X} := \mathcal{E}^{\text{info}}(F, p) \setminus \mathcal{N}.$$

We now ask whether existence or nonexistence of FED is topologically generic. The key observation is that, under a full-support prior, full-support posterior distributions are topologically large in the reduced-form space $\mathcal{E}(F, p)$. By Remark C.1, such experiments never admit a FED equilibrium at $b = 0$ whenever they are informative.

To formalize this idea, let $\mathcal{I}_{\mathbb{Q}}$ denote the set of open intervals in $[0, 1]$ with rational endpoints. For each $i \in \{1, \dots, N\}$ and each $I \in \mathcal{I}_{\mathbb{Q}}$, define

$$U_{i,I} := \{E = (F_1, \dots, F_N) \in \mathcal{E}(F, p) : F_i(I) > 0\}.$$

Thus $U_{i,I}$ is the set of experiments for which the i-th posterior assigns positive probability to the interval I. Since rational open intervals form a countable basis of the topology on $[0, 1]$, requiring membership in all such sets $U_{i,I}$ is exactly the same as requiring that every posterior F_i have full support on $[0, 1]$.

The next result shows that these full-support experiments are residual, and hence that nonexistence of FED is topologically generic.

Lemma C.3. *The set $\mathcal{N}$ is residual in $\mathcal{E}^{\text{info}}(F, p)$ and the set $\mathcal{X}$ is meagre.*[10]

Proof. We proceed in three steps.

Step 1: each set $U_{i,I}$ is open and dense in $\mathcal{E}(F, p)$. Fix $i \in \{1, \dots, N\}$ and $I \in \mathcal{I}_{\mathbb{Q}}$. We first claim that $U_{i,I}$ is open. Since I is open, the map $\mu \mapsto \mu(I)$ is lower semicontinuous under weak convergence on $\Delta([0, 1])$. Therefore the set $\{\mu \in \Delta([0, 1]) : \mu(I) > 0\}$ is open. Because $\mathcal{E}(F, p)$ carries the subspace topology inherited from the product weak topology on $\Delta([0, 1])^N$, it follows that $U_{i,I}$ is open in $\mathcal{E}(F, p)$.

We next claim that $U_{i,I}$ is dense. Fix any experiment $E = (F_1, \dots, F_N) \in \mathcal{E}(F, p)$. For $\varepsilon \in (0, 1)$, define perturbed posteriors

$$F_i^{\varepsilon} := (1 - \varepsilon)F_i + \varepsilon F, \qquad i = 1, \dots, N.$$

Then $\sum_{i=1}^N p_i F_i^{\varepsilon} = (1 - \varepsilon) \sum_{i=1}^N p_i F_i + \varepsilon \sum_{i=1}^N p_i F = (1 - \varepsilon)F + \varepsilon F = F$, so $E^{\varepsilon} := (F_1^{\varepsilon}, \dots, F_N^{\varepsilon}) \in \mathcal{E}(F, p)$. Moreover, because the prior F has full support, every nonempty open interval I satisfies $F(I) > 0$. Hence $F_i^{\varepsilon}(I) = (1 - \varepsilon)F_i(I) + \varepsilon F(I) \geq \varepsilon F(I) > 0$, and therefore $E^{\varepsilon} \in U_{i,I}$.

[10] For the definitions of residual sets and meagre sets, see page 41 of Kechris (1995).

Finally, $E^\varepsilon \to E$ in the product weak topology as $\varepsilon \downarrow 0$. Thus every neighborhood of E intersects $U_{i,I}$, which proves that $U_{i,I}$ is dense.

Step 2: the set of full-support experiments is residual. Define

$$\mathcal{R}_{\text{fs}} := \bigcap_{i=1}^{N} \bigcap_{I \in \mathcal{I}_{\mathbb{Q}}} U_{i,I}.$$

Since $\mathcal{I}_{\mathbb{Q}}$ is countable and each $U_{i,I}$ is open and dense, the set $\mathcal{R}_{\text{fs}}$ is residual in $\mathcal{E}(F, p)$.

We now identify $\mathcal{R}_{\text{fs}}$. By construction, an experiment $E = (F_1, \ldots, F_N)$ belongs to $\mathcal{R}_{\text{fs}}$ if and only if for every i and every rational open interval I, $F_i(I) > 0$. Since rational open intervals form a basis for the topology of $[0, 1]$, this is equivalent to requiring that every nonempty open interval have positive F_i-mass, which in turn is equivalent to $\operatorname{supp}(F_i) = [0, 1]$ for all i. Thus $\mathcal{R}_{\text{fs}}$ is exactly the set of full-support experiments.

Step 3: conclusion. By Remark C.1, if $E \in \mathcal{E}^{\text{info}}(F, p)$ and $\operatorname{supp}(F_i) = [0, 1]$ for every i, then no FED equilibrium exists at $b = 0$. In other words, $\mathcal{R}_{\text{fs}} \cap \mathcal{E}^{\text{info}}(F, p) \subseteq \mathcal{N}$. Since $\mathcal{R}_{\text{fs}}$ is residual in $\mathcal{E}(F, p)$, the set $\mathcal{R}_{\text{fs}} \cap \mathcal{E}^{\text{info}}(F, p)$ is residual in the subspace $\mathcal{E}^{\text{info}}(F, p)$. Hence $\mathcal{N}$ contains a residual subset of $\mathcal{E}^{\text{info}}(F, p)$, and therefore $\mathcal{N}$ itself is residual in $\mathcal{E}^{\text{info}}(F, p)$. Consequently its complement $\mathcal{X} = \mathcal{E}^{\text{info}}(F, p) \setminus \mathcal{N}$ is meagre. □

Lemma C.3 shows that, in the reduced-form space of Bayes-plausible posterior tuples, nonexistence of FED at $b = 0$ is the generic outcome. In contrast, the set of experiments for which FED exists at $b = 0$ is topologically small. As an immediate corollary, the class of deterministic experiments is meagre.

Remark C.4 (Robustness of generic nonexistence to general evidence structures)**.** *The argument above fixes the signal probabilities $p_1, \ldots, p_N$ and varies only the posterior distributions $(F_1, \ldots, F_N)$. This restriction is only for expositional convenience. The same genericity conclusion continues to hold if the probabilities p_i are also allowed to vary.*

More generally, the result does not rely on the signal space being finite. An arbitrary evidence structure can be represented by the distribution of posterior beliefs induced by the experiment, that is, by a probability measure η on $\Delta([0, 1])$ satisfying the Bayes plausibility condition

$$\int_{\Delta([0,1])} \mu \, d\eta(\mu) = F.$$

The finite-N representation corresponds to the special case in which η has finite support, namely $\eta = \sum_{i=1}^{N} p_i \delta_{F_i}$.

The same perturbation argument used in the proof of Lemma C.3 applies in this more general setting: by mixing each posterior with the prior F, one can approximate any experiment by experiments whose posterior distributions have full support. Since informative experiments with full-support posteriors do not admit FED at $b = 0$ (Remark C.1), the set of experiments for which FED fails at $b = 0$ remains residual in the general space of evidence structures.

Remark C.5 (Robustness of generic nonexistence to richer message structures)**.** *The generic nonexistence result above was derived under the minimal verifiable message structure $M(s_i) = \{s_i, \circ\}$. This conclusion remains robust when the sender is allowed a richer set of verifiable messages.*

The key observation follows from Lemma C.1. If a candidate assessment fails to be an equilibrium under a given message structure $M(\cdot)$, it must also fail under any richer message structure $M'(\cdot)$, since every deviation feasible under $M(\cdot)$ remains feasible under $M'(\cdot)$. In particular, if the FED profile cannot be sustained under the minimal verifiable structure, the same profitable deviations remain available when additional messages are allowed.

Combining this observation with Lemma C.3 implies that the generic nonexistence result extends to richer message structures. Even if the sender can use a larger set of verifiable statements, the set of experiments for which a FED equilibrium exists at $b = 0$ remains meagre, and the nonexistence of FED continues to be topologically generic.

C.3 Deterministic Evidence

We consider a model of deterministic evidence. Fix $N \geq 2$. Let $C = \{C_1, \ldots, C_N\}$ be a monotone interval partition of Θ. That is, Let $0 = c_0 < c_1 < \cdots < c_N = 1$ and define a monotone interval partition $C_i = [c_{i-1}, c_i)$ for $i = 1, \ldots, N-1$, $\quad C_N = [c_{N-1}, 1]$. Let $\pi : \Theta \to \Delta(S)$ such that $\pi(s_i|\theta) = 1$ if $\theta \in C_i$ and $\pi(s_i|\theta) = 0$ otherwise. Note that $\mathbb{E}(\theta|s_i) > \mathbb{E}(\theta|s_j)$ for all $i > j$. When such partition C exists that characterizes the evidence structure (S, π), we say that the evidence is deterministic.

Proposition C.1. *Suppose that the evidence structure is deterministic. An FED equilibrium exists for all b.*

Proof. Suppose that all (θ, s)-type of senders discloses the signal with probability 1. Let $a_i \triangleq \mathbb{E}(\theta \mid s_i)$ for all i. Denote $a_0 \triangleq \mathbb{E}(\theta \mid \circ)$. By construction, $a_1 < a_2 < \cdots < a_N$.

Fix an index $k \in \{1, \ldots, N\}$. Consider the candidate full-disclosure strategies and set the off-path action $a_0 = a_i$. Since evidence is verifiable, the only deviation for a type $\theta \in C_i$ is from s_i to $\circ$. If $i < k$ then $a_i < a_k$ and the sender discloses iff

$$c_i + b \leq \frac{a_i + a_k}{2}. \tag{IC-L$_{i,k}$}$$

If $k < i$ then $a_k < a_i$ and the sender discloses iff

$$c_{i-1} + b \geq \frac{a_i + a_k}{2}. \tag{IC-R$_{i,k}$}$$

If $i = k$ then $a_i = a_k$ and every type in C_i is indifferent between disclosing and remaining silent. Define the set of left-feasible indices

$$L = \Big\{ k \in \{1, \ldots, N\} : \text{for all } i < k,\ \text{IC-L}_{i,k} \text{ holds} \Big\}.$$

Since the condition is vacuous for $k = 1$, we have $1 \in L$, so L is nonempty. Let $k^* = \max L$. We claim that k^* also satisfies all right constraints (IC-R$_{i,k^*}$) for $i > k^*$. To see why, suppose that it not the case. Then there exists some $i > k^*$ such that $c_{i-1} + b < \frac{a_i + a_{k^*}}{2}$. Let $i^* > k^*$ be the smallest index for which this strict inequality holds. Rearranging, we obtain

$$a_{i^*} > 2(c_{i^*-1} + b) - a_{k^*}. \tag{8}$$

We show that $i^* \in L$, contradicting the maximality of k^*. Fix any $j < i^*$. If $j < k^*$, then since $k^* \in L$ we have $c_j + b \leq \frac{a_j + a_{k^*}}{2}$. Because $i^* > k^*$ and $a_{i^*} > a_{k^*}$, it follows that

$\frac{a_j+a_{k^*}}{2} < \frac{a_j+a_{i^*}}{2}$, so $c_j + b \leq \frac{a_j+a_{i^*}}{2}$. Now consider $k^* \leq j < i^*$. Using (8), monotonicity of cutpoints ($c_j \leq c_{i^*-1}$), and monotonicity of actions ($a_j \geq a_{k^*}$), we have

$$2(c_j + b) - a_j \leq 2(c_{i^*-1} + b) - a_{k^*} < a_{i^*}.$$

Rearranging yields $c_j + b < \frac{a_j+a_{i^*}}{2}$. Therefore, for every $j < i^*$ we have

$$c_j + b \leq \frac{a_j + a_{i^*}}{2},$$

which shows $i^* \in L$. But $i^* > k^* = \max L$, a contradiction. Hence k^* satisfies all right constraints (IC-R$_{i,k^*}$) for $i > k^*$.

Therefore, k^* satisfies (IC-L$_{i,k^*}$) for all $i < k^*$ and (IC-R$_{i,k^*}$) for all $i > k^*$, and types in C_{k^*} are indifferent. Hence every type weakly prefers disclosing s_i to deviating to $\circ$. Therefore, a FED equilibrium exists. □

This result stands in stark contrast to Proposition 2, which shows that FED does not exist when b is small. What is particularly interesting is that our baseline model, with noisy full-support evidence, can approximate arbitrarily well any model of deterministic evidence. The result above therefore points to a sharp discontinuity. Consider a sequence of evidence structures satisfying our baseline assumption that converges to a deterministic evidence structure. Along the sequence, FED does not exist for small b; in the limit, it does. Note that the threshold $\bar{b}$ depends on π, through $E(\theta \mid s_N)$. However, I suspect that one can construct a sequence along which the threshold remains constant, or at the very least is bounded away from 0.

Both the deterministic and noisy evidence models capture the idea that the sender knows more than she can prove. Indeed, in both models the sender observes θ. One can also construct deterministic and noisy evidence structures that generate the same posterior means, so in that sense the two structures can be equally informative. What changes is the noise, which affects which θ-types can disclose, with positive probability, which signals. Under deterministic evidence, there is no state θ such that the sender can send two different signals. This clarifies that the key ingredient behind the failure of the unraveling principle is not merely that the sender knows more than she can prove, but rather that evidence is noisy in the sense described above. The deterministic-evidence result is therefore non-robust. A tiny amount of noise can eliminate FED. This is interesting because it suggests that deterministic evidence structures are extreme. It remains an open question which of our results extend under deterministic evidence.

C.4 A More General Model

We write a more general model that captures in a more genuine way the idea that "the sender knows more than what her evidence can prove." We then prove that our baseline model from Section 2 is strategically equivalent to this more general model (net of the assumptions on F and π that may distinguish between deterministic and noisy evidence).

The Ex Ante Model. Let the true payoff-relevant state be $\vartheta \in [0,1]$, distributed according to a cumulative distribution function Q with full support on $[0,1]$. Let T be an arbitrary metric space (endowed with its Borel σ-algebra) and let $S = \{s_1, \ldots, s_N\}$ be a finite evidence set. Consider an information structure $\psi : [0,1] \to \Delta(T \times S)$. A sender privately observes the realized pair (t,s) drawn from $\psi(\cdot|\vartheta)$. The component t is *unverifiable*, while s is *verifiable*. Formally, this means that the message set for the sender of type (t,s) is $M(s) \triangleq \{\circ, s\}$, for all t. That is, a (behavioral) sender strategy is a measurable map $\sigma : T \times S \to \Delta(M)$ satisfying the feasibility constraints $\sigma(m|t,s) = 0$ for all $m \notin M(s)$. A receiver strategy is a map $\alpha : M \to [0,1]$.

The prior Q and the information structure (T, S, ψ) induce a joint law P on $[0,1] \times T \times S$ given by $P(d\vartheta, dt, ds) = Q(d\vartheta)\psi(dt, ds|\vartheta)$. Whenever Pr is used below, it refers to this joint law or its conditionals.

From Ex Ante to Ad Interim. Define the *interim state* $\theta(t,s) \triangleq \mathbb{E}[\vartheta|t,s]$ for all (t,s). Let the induced interim state space be the image of this map on the support of (t,s):

$$\Theta \triangleq \{\theta(t,s) : (t,s) \in \operatorname{supp}(P_{t,s})\} \subseteq [0,1].$$

Let F be the distribution of θ induced by (Q, ψ), that is, for all $x \in [0,1]$, $F(x) \triangleq \Pr(\theta \leq x)$. Finally, define the induced evidence structure $\pi : \Theta \to \Delta(S)$ with $\pi(s_i|\theta) \triangleq \Pr(s = s_i|\theta)$ for all i. Therefore, we have mapped this more general model in the language of our baseline model which is parametrized by (Θ, F, S, π).

Equivalence. Next, we argue the interim model and the ex ante model are strategically equivalent. The key observation is that the receiver's optimal action depends on the induced posterior mean of ϑ, and the sender's message incentives depend on (t,s) only through $\theta(t,s) = \mathbb{E}[\vartheta|t,s]$. First, for any message $m \in M$, the receiver's best reply in the ex ante model is $\alpha(m) = \mathbb{E}[\vartheta|m] = \mathbb{E}[\mathbb{E}[\vartheta|t,s] \mid m] = \mathbb{E}[\theta \mid m]$. Thus, the receiver's best reply depends on the induced conditional distribution of θ given m only, which is also the receiver's best reply in the interim model. Second, fix any receiver strategy α and consider the sender's interim expected payoff in the ex ante model conditional on (t,s). Let $\theta = \theta(t,s)$. Then

$$\mathbb{E}[u_S(\alpha(m),\vartheta) \mid t,s] = -\mathbb{E}[(\alpha(m)-\vartheta-b)^2 \mid t,s] = -\operatorname{Var}(\vartheta|t,s) - (\alpha(m)-\theta-b)^2.$$

Since the term $\operatorname{Var}(\vartheta|t,s)$ does not depend on m, the sender's behavior depends on (t,s) only through θ. But this is exactly the sender payoff comparison in the interim model.

C.5 All Signals Disclosed on Path

The following result shows that it is without loss of generality to focus on PBEs under which all signals are disclosed with positive probability. The construction relies on setting some $\alpha(s_i)$ equal to $\alpha(\circ)$.

Lemma C.4. *Fix any PBE. Then there exists a (possibly different) PBE that is outcome equivalent such that every signal $s_i \in S$ is disclosed with strictly positive probability.*

Proof. Let $(\sigma^*, \alpha^*, \mu^*)$. If $\Pr_{\sigma^*}(m = s_i) > 0$ for all i, we are done. Suppose instead that the set of signals whose disclosure message is off path is nonempty. Denote this set by J. Then for all $i \in J$ and for F-almost all θ, $\sigma^*(\circ|\theta, s_i) = 1$ and, thus, $m = \circ$ is on path. Let $a_0 \triangleq \alpha^*(\circ) = E_{\mu^*}[\theta|\circ]$ and note $a_0 \in (0,1)$. (Indeed, if $a_0 = 0$ then $\theta = 0$ almost surely conditional on $m = \circ$ (since $\theta \geq 0$), which is impossible because θ is atomless on $[0,1]$ and $\Pr(m = \circ) > 0$; similarly $a_0 \neq 1$.)

Fix any $i \in J$. For $t \in (0,1]$ let

$$L_i(t) \triangleq \mathbb{E}(\theta|s_i, \theta \leq t) \qquad \text{and} \qquad H_i(t) \triangleq \mathbb{E}(\theta|s_i, \theta \geq t)$$

Set $L_i(0) \triangleq 0$ and $H_i(1) \triangleq 1$. Note that L_i and H_i are continuous and that $L_i(1) = H_i(0)$. Therefore, since $a_0 \in (0,1)$, at least one of the following holds:

- $a_0 \leq E[\theta|s_i]$, in which case by the intermediate value theorem there exists $t_i \in (0,1]$ such that $L_i(t_i) = a_0$;
- $a_0 \geq E[\theta|s_i]$, in which case there exists $t_i \in [0,1)$ such that $H_i(t_i) = a_0$.

In either case we can choose a measurable set $B_i \subseteq [0,1]$ of positive probability conditional on s_i such that $\mathbb{E}[\theta \mid s = s_i,\ \theta \in B_i] = a_0$.

Define a new sender strategy σ' as follows. If $i \notin J$, set $\sigma'(\cdot|\theta, s_i) = \sigma^*(\cdot|\theta, s_i)$ for all θ. If instead $i \in J$, set $\sigma'(s_i|\theta, s_i) = \mathbb{1}\{\theta \in B_i\}$. Define the new receiver strategy α' by $\alpha'(s_i) = \alpha^*(s_i)$ if $i \notin J$, and $\alpha'(\circ) = \alpha'(s_i) = a_0$ otherwise. Let μ' be given by Bayes' rule on every on-path message under σ', and arbitrary off path (if any).

We need to verify that (σ', α', μ') is a PBE. Let's start from the receiver optimality. Trivially, it is satisfied for any message $m = s_i$ with $i \notin J$. Moreover, if $m = s_i$ with $i \in J$, we have that by construction:

$$\mathbb{E}_{\mu'}[\theta|s_i] = \mathbb{E}[\theta \mid s = s_i,\ \theta \in B_i] = a_0 = \alpha'(s_i).$$

Finally, consider $m = \circ$. Let $\Omega_0 \triangleq \{(\theta, s) : m = \circ \text{ under } \sigma^*\}$ and

$$B \triangleq \bigcup_{i \in J} \{(\theta, s_i) : \theta \in B_i\}.$$

By construction, for $i \in J$ we had $\sigma^*(\circ|\theta, s_i) = 1$, so $B \subseteq \Omega_0$. Moreover, we have $\mathbb{E}[\theta|B] = a_0$ and $\mathbb{E}[\theta|\Omega_0] = a_0$. Under σ', the event $\{m = \circ\}$ is exactly $\Omega_0 \setminus B$. Thus,

$$\mathbb{E}[\theta|\Omega_0] = \Pr(B|\Omega_0)\mathbb{E}[\theta|B] + \Pr(\Omega_0 \setminus B|\Omega_0)\mathbb{E}[\theta|\Omega_0 \setminus B].$$

Thus, $\mathbb{E}[\theta|\Omega_0 \setminus B] = a_0$. Therefore, under σ', $\mathbb{E}_{\mu'}[\theta|\circ] = a_0 = \alpha'(\circ)$. Thus α' is sequentially rational after every on-path message. It is immediate to verify sender's optimality and belief consistency. □

C.6 Example of More Informative Evidence

We present another example to show that Blackwell more informative evidence may lead to efficient equilibria that are less informative. Suppose that F is uniform, $b = \frac{1}{4}$, and $N = 2$. We consider two evidence structures, (S, π) and $(S, \hat{\pi})$. The former is such that $\pi(s_2|\theta) = \frac{1}{2}\theta$ and $\pi(s_1|\theta) = 1 - \pi(s_2|\theta)$. The latter is $\hat{\pi}(s_2|\theta) = \frac{1}{4}$ and $\hat{\pi}(s_1|\theta) = \frac{3}{4}$. Note that (S, π) is strictly informative. The conditional posterior means are $\mathbb{E}(\theta|s_1) = \frac{4}{9}$ and $\mathbb{E}(\theta|s_2) = \frac{2}{3}$. Conversely, $(S, \hat{\pi})$ uninformative and for all i, $\hat{\pi}(s_i|\theta) = \Pr_\pi(s_i) = \int \pi(s_i|\theta)d\theta$. Therefore, the unconditional probability distribution of the signal is the same in the two cases. This example is interesting because in a way the two cases differ only in so far as the signals are informative or not. They do not differ in the probability a signal can be sent or not.

In this example, there is an equilibrium in the disclosure game with uninformative evidence that is strictly more informative than the most informative equilibrium in the disclosure game with informative evidence. Therefore, when the evidence structure is Blackwell more informative, the efficient equilibrium is not necessarily more informative.

By Lemma 1, it is without loss of generality to assume that equilibria have a generalized cutoff form, with partition (L, M, H) and thresholds $t \in [0,1]^{|L \cup H|}$. We claim that every nonbabbling equilibrium outcome must have $L = \emptyset$ and is therefore one of the following three candidates: (i) $H = \{s_2\}$; (ii) $H = \{s_1\}$; (iii) $H = \{s_1, s_2\}$. Among these, the former is the most informative.

There are $3^2 = 9$ possible partitions of $\{1,2\}$ into L, M, H. The babbling equilibrium ($M = \{1,2\}$) exists but is uninformative, so we ignore it without loss. We show next that no non-babbling equilibrium can have $L \neq \emptyset$. In the following, whenever we fix an equilibrium we write $a_0 = \alpha(\emptyset)$ and $a_i = \alpha(s_i)$ for $i \in \{1,2\}$.

Case 1. $L = \{2\}$**,** $M = \{1\}$. Suppose s_2 is disclosed iff $\theta \le t$. Then $a_2(t) = \mathbb{E}[\theta | s_2, \theta \le t] = \frac{2}{3}t$. The disclosure probability and its first moment are

$$x_2(t) = \int_0^t \frac{\theta}{2} d\theta = \frac{t^2}{4}, \qquad y_2(t) = \int_0^t \theta \frac{\theta}{2} d\theta = \frac{t^3}{6}.$$

Since $\mathbb{E}[\theta] = \frac{1}{2}$, the silence posterior mean is

$$a_0(t) = \mathbb{E}[\theta|\emptyset] = \frac{\frac{1}{2} - y_2(t)}{1 - x_2(t)} = \frac{\frac{1}{2} - \frac{t^3}{6}}{1 - \frac{t^2}{4}} = \frac{2}{3}\frac{3 - t^3}{4 - t^2}.$$

The cutoff condition $t = \frac{a_2(t) + a_0(t)}{2} - \frac{1}{4}$ simplifies to $t(4t^2 + 3t - 32) = 0$. the only solution in $[0,1]$ is $t = 0$, which leads to a babbling equilibrium, a contradiction.

Case 2. $L = \{1\}$**,** $M = \{2\}$**.** Suppose s_1 is disclosed iff $\theta \le t$. Then $a_1(t) = \mathbb{E}[\theta | s_1, \theta \le t] = \frac{\frac{t^2}{2} - \frac{t^3}{6}}{t - \frac{t^2}{4}}$. The disclosure probability and its first moment are

$$x_1(t) = \int_0^t (1 - \frac{\theta}{2}) d\theta = t - \frac{t^2}{4}, \qquad y_1(t) = \int_0^t \theta(1 - \frac{\theta}{2}) d\theta = \frac{t^2}{2} - \frac{t^3}{6}.$$

Thus

$$a_0(t) = \frac{\frac{1}{2} - y_1(t)}{1 - x_1(t)} = \frac{\frac{1}{2} - \frac{t^2}{2} + \frac{t^3}{6}}{1 - t + \frac{t^2}{4}}.$$

The cutoff condition $t = \frac{a_1(t) + a_0(t)}{2} - \frac{1}{4}$ reduces to $t(4t^3 - 37t^2 + 104t - 96) = 0$. Again, the only solution in $[0,1]$ is $t = 0$, which leads to a babbling equilibrium, a contradiction.

Case 3. $L = \{1,2\}$**.** Let t_1, t_2 be such that s_1 is disclosed iff $\theta \le t_1$ and s_2 iff $\theta \le t_2$. The indifference conditions imply that the silence action a_0 must satisfy

$$a_0 = 2\Big(t_1 + \frac{1}{4}\Big) - a_1(t_1), \qquad a_0 = 2\Big(t_2 + \frac{1}{4}\Big) - a_2(t_2),$$

where $a_1(t_1)$ and $a_2(t_2)$ are as in Cases 1 and 2. Eliminating a_0 yields $t_2 = \frac{t_1(9-2t_1)}{2(4-t_1)}$. Imposing Bayes consistency for a_0 and substituting this relation reduces equilibrium feasibility to

$$t_1\Big(72t_1^4 - 1040t_1^3 + 5297t_1^2 - 10700t_1 + 6144\Big) = 0.$$

The quartic factor has a unique root in $(0,1)$ at $t_1 \approx 0.9275$, but for any $t_1 > \frac{11-\sqrt{57}}{4} \approx 0.8625$ the implied $t_2 > 1$, violating admissibility. Hence the only admissible solution is $t_1 = t_2 = 0$, which leads to a babbling equilibrium, a contradiction.

Case 4. $L = \{1\}$**,** $H = \{2\}$**.** Let t_1, t_2 be the cutoffs. The silence message pools the events $\{(s_1, \theta > t_1)\} \cup \{(s_2, \theta < t_2)\}$. Write $\mu_1^{hi}(t_1) \triangleq \mathbb{E}[\theta | s_1, \theta > t_1]$ and $\mu_2^{lo}(t_2) \triangleq \mathbb{E}[\theta | s_2, \theta < t_2] = \frac{2}{3}t_2$. By Bayes rule, a_0 is a convex combination of $\mu_1^{hi}(t_1)$ and $\mu_2^{lo}(t_2)$, hence

$$a_0 \in [\min\{\mu_1^{hi}(t_1), \mu_2^{lo}(t_2)\}, \max\{\mu_1^{hi}(t_1), \mu_2^{lo}(t_2)\}].$$

From $s_1 \in L$, sender indifference implies

$$a_0 = 2\Big(t_1 + \frac{1}{4}\Big) - \mu_1^{lo}(t_1), \qquad \text{where } \mu_1^{lo}(t_1) \triangleq \mathbb{E}[\theta | s_1, \theta < t_1].$$

A direct algebraic comparison yields, for all $t_1 \in (0,1)$,

$$2\Big(t_1 + \frac{1}{4}\Big) - \mu_1^{lo}(t_1) - \mu_1^{hi}(t_1) = \frac{4t_1^3 - 33t_1^2 + 63t_1 + 4}{6(4-t_1)(3-t_1)} > 0,$$

so $a_0 > \mu_1^{hi}(t_1)$. Since a_0 is a convex combination of $\mu_1^{hi}(t_1)$ and $\mu_2^{lo}(t_2)$, this forces $\mu_2^{lo}(t_2) > a_0$. In particular $\mu_2^{lo}(t_2) > \mu_1^{hi}(t_1)$. Moreover, $\mu_1^{hi}(t_1) \ge \mathbb{E}[\theta | s_1] = \frac{4}{9}$, because

$$\mu_1^{hi}(t) - \frac{4}{9} = \frac{2t(8-3t)}{9(3-t)} \ge 0.$$

Therefore, $\frac{2}{3}t_2 = \mu_2^{lo}(t_2) > \frac{4}{9}$ implies $t_2 > \frac{2}{3}$. On the other hand, since $\mu_2^{lo}(t_2) > a_0$ and $s_2 \in H$, sender indifference at t_2 implies

$$a_0 = 2\Big(t_2 + \frac{1}{4}\Big) - \mu_2^{hi}(t_2), \qquad \mu_2^{hi}(t_2) = \mathbb{E}[\theta | s_2, \theta > t_2] = \frac{2}{3}\frac{1 + t_2 + t_2^2}{1 + t_2}.$$

Thus $\mu_2^{lo}(t_2) > a_0$ is equivalent to

$$2\Big(t_2 + \frac{1}{4}\Big) - \mu_2^{hi}(t_2) < \frac{2}{3}t_2,$$

or

$$\frac{4t_2^2 + 7t_2 - 1}{6(1+t_2)} < 0 \qquad \Longrightarrow \qquad t_2 < \frac{-7+\sqrt{65}}{8}.$$

Since $\frac{-7+\sqrt{65}}{8} < \frac{1}{6}$, this contradicts $t_2 > \frac{2}{3}$. Hence no equilibrium exists with $L = \{1\}$, $H = \{2\}$.

Case 5. $H = \{1\}$**,** $L = \{2\}$**.** Let t_1, t_2 be the cutoffs. The two indifference conditions imply

$$a_0 = 2\Big(t_1 + \frac{1}{4}\Big) - \mathbb{E}[\theta|s_1, \theta > t_1], \qquad a_0 = \frac{1}{2} + \frac{4}{3}t_2.$$

Eliminating a_0 yields an explicit relation $t_2 = \frac{t_1^2 - \frac{7}{2}t_1 + 1}{t_1 - 3}$. For $t_2 \geq 0$ with $t_1 \in (0,1)$ we must have $t_1 \geq \frac{7-\sqrt{33}}{4}$. Imposing Bayes consistency for a_0 and substituting the above relation reduces feasibility to a single polynomial equation in t_1:

$$32t_1^6 - 384t_1^5 + 1672t_1^4 - 2975t_1^3 + 1183t_1^2 + 1548t_1 - 416 = 0.$$

This polynomial has a unique root in $(0,1)$, namely $t_1 \approx 0.2473$, which is strictly smaller than $\frac{7-\sqrt{33}}{4}$. Therefore any $(0,1)$-root implies $t_2 < 0$, contradicting admissibility. Hence no equilibrium exists with $H = \{1\}$, $L = \{2\}$.

Therefore, every nonbabbling equilibrium must have $L = \emptyset$. Therefore, the only possible nonbabbling partitions are: $(H = \{2\}, M = \{1\})$; $(H = \{1\}, M = \{2\})$; $(H = \{1,2\})$.

Case I: $(H = \{2\}, M = \{1\})$**.** Here s_2 is disclosed iff $\theta \geq t$, and s_1 is off-path. Bayes' rule gives $\Pr(m = s_2) = \int_t^1 \frac{\theta}{2} d\theta = \frac{1-t^2}{4}$,

$$a_2(t) = \mathbb{E}[\theta|s_2, \theta \geq t] = \frac{\int_t^1 \theta \frac{\theta}{2} d\theta}{\int_t^1 \frac{\theta}{2} d\theta} = \frac{2}{3}\frac{1+t+t^2}{1+t},$$

$$a_0(t) = \mathbb{E}[\theta|\emptyset] = \frac{\frac{1}{2} - \int_t^1 \theta \frac{\theta}{2} d\theta}{1 - \int_t^1 \frac{\theta}{2} d\theta} = \frac{2}{3}\frac{2+t^3}{3+t^2}.$$

The cutoff condition $t = \frac{a_2(t)+a_0(t)}{2} - \frac{1}{4}$ is equivalent to

$$4t^4 + 7t^3 + 23t^2 + 25t - 11 = 0,$$

which has a unique root in $(0,1)$, $t^{ST2} \approx 0.33$. Let $q_2 = \Pr(m = s_2) = \frac{1-(t^{ST2})^2}{4}$ and $q_0 = 1 - q_2$. Then $V_I = q_2 a_2^2 + q_0 a_0^2 - (\frac{1}{2})^2 \approx 0.014$.

Case II: $(H = \{1\}, M = \{2\})$. Here s_1 is disclosed iff $\theta \geq t$, and s_2 is off-path. Compute

$$q_1 = \Pr(m = s_1) = \int_t^1 \left(1 - \frac{\theta}{2}\right) d\theta = \frac{3}{4} - t + \frac{t^2}{4},$$

$$a_1(t) = \mathbb{E}[\theta | s_1, \theta \geq t] = \frac{\int_t^1 \theta(1 - \frac{\theta}{2}) d\theta}{\int_t^1 (1 - \frac{\theta}{2}) d\theta} = \frac{\frac{1}{3} - \frac{t^2}{2} + \frac{t^3}{6}}{\frac{3}{4} - t + \frac{t^2}{4}},$$

$$a_0(t) = \mathbb{E}[\theta | \varnothing] = \frac{\frac{1}{2} - \int_t^1 \theta(1 - \frac{\theta}{2}) d\theta}{1 - \int_t^1 (1 - \frac{\theta}{2}) d\theta} = \frac{\frac{1}{6} + \frac{t^2}{2} - \frac{t^3}{6}}{\frac{1}{4} + t - \frac{t^2}{4}}.$$

The cutoff condition $t = \frac{a_1(t) + a_0(t)}{2} - \frac{1}{4}$ is equivalent to

$$4t^4 - 33t^3 + 55t^2 + 33t - 11 = 0,$$

which has a unique root in $(0,1)$, $t^{ST1} \approx 0.25$. With $q_0 = 1 - q_1$, informativeness is $V_{II} = q_1 a_1^2 + q_0 a_0^2 - (\frac{1}{2})^2 \approx 0.009$.

Case III: Here s_1 is disclosed iff $\theta \geq t_1$ and s_2 iff $\theta \geq t_2$. Define

$$q_1 = \int_{t_1}^1 \left(1 - \frac{\theta}{2}\right) d\theta = \frac{3}{4} - t_1 + \frac{t_1^2}{4}, \qquad q_2 = \int_{t_2}^1 \frac{\theta}{2} d\theta = \frac{1 - t_2^2}{4}, \qquad q_0 = 1 - q_1 - q_2,$$

$$a_1(t_1) = \frac{\frac{1}{3} - \frac{t_1^2}{2} + \frac{t_1^3}{6}}{\frac{3}{4} - t_1 + \frac{t_1^2}{4}}, \qquad a_2(t_2) = \frac{2}{3} \frac{1 + t_2 + t_2^2}{1 + t_2},$$

$$a_0(t_1, t_2) = \frac{\frac{1}{2} - \int_{t_1}^1 \theta(1 - \frac{\theta}{2}) d\theta - \int_{t_2}^1 \theta \frac{\theta}{2} d\theta}{1 - q_1 - q_2} = \frac{2}{3} \frac{3t_1^2 - t_1^3 + t_2^3}{4t_1 - t_1^2 + t_2^2}.$$

The cutoff conditions are

$$t_1 = \frac{a_1(t_1) + a_0(t_1, t_2)}{2} - \frac{1}{4}, \qquad t_2 = \frac{a_2(t_2) + a_0(t_1, t_2)}{2} - \frac{1}{4},$$

which admit a unique solution in $(0,1)^2$: $t_1^{HH} \approx 0.001$ and $t_2^{HH} \approx 0.116$. Thus, $V_{HH} = q_1 a_1^2 + q_2 a_2^2 + q_0 a_0^2 - \left(\frac{1}{2}\right)^2 \approx 0.011$.

Therefore, among all the equilibria of the game under π, the efficient one is Case II and it leads to an informativeness of $V_\pi \approx 0.014$.

To Do. Compute "single top" eqm under $\hat{\pi}$ and show it is more informative than 0.014. This is straighforward I'll leave it to later.